Charged particle transport in 3-dimensional stochastic magnetic fields

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Particle transport in stochastic media is an important topic in astrophysics [1]. A variety of problems, such as low-energy cosmic ray penetration into the heliosphere, the propagation of galactic cosmic rays in and out of the interstellar magnetic field, anomalous escape rates of runaways in the atmosphere are directly related to charged particle motion in fluctuating magnetic fields. This is a complex process due to the Lagrangian non-linearity determined by the space-dependence of the stochastic magnetic field. An important progress was obtained in the last decade due to the development of statistical methods that permitted to go beyond the quasilinear regime. Semi-analytical statistical approaches: the decorrelation trajectory method [2] and the nested subensemble approach [3] were developed in studies of transport in magnetically confined plasmas. In this context, transport in magnetic turbulence was studied in [4]. It was shown that trapping of the magnetic lines generates stochastic magnetic islands, that appear as quasicoherent solenoidal structures. These stochastic magnetic islands strongly influence magnetic line and particle transport.

The aim of this paper is to determine the nonlinear effects that appear in charged particle transport in magnetic turbulence for conditions relevant to space plasmas. The parameters of the stochastic magnetic field for the space and fusion plasmas are completely different. However, the fundamental difference between the two systems is not due to the parameters but to the configuration of the magnetic field. In fusion plasmas there is a very large average magnetic field while very small average magnetic field, of the same order as the fluctuating component, exists in space plasmas. Consequently, 2-dimensional stochastic models and guiding center approximation can be used in fusion plasmas. Lorentz force and 3-dimensional description of the transport is necessary for space plasmas, which makes the model much more complicated than that of fusion plasmas.

2. The model

We consider a stochastic magnetic field $\mathbf{B_t}=B_0\mathbf{e_z}+\mathbf{B}$ where B_0 is a constant average component directed along z-axis, and $\mathbf{B}$ is the stochastic component that depends on $\mathbf{x}=(x_1, x_2)$ and z. The stochastic component has an arbitrary direction. Its structure is taken similar to the model in the numerical simulation presented in [5], which corresponds to the potential vector of the type $\boldsymbol{A}=[-\partial_2 A^{(2)}, \partial_1 A^{(2)}, A^{(1)}]$ where $A^{(1)}$, $A^{(2)}$ are scalar fields and ∂_i is the derivative with respect to x_i. The magnetic field is

$$\begin{aligned} B_1 &= \partial_2 A^{(1)} - \partial_1 \partial_z A^{(2)}, \\ B_2 &= -\partial_1 A^{(1)} - \partial_2 \partial_z A^{(2)}, \\ B_z &= \partial_1{}^2 A^{(2)} + \partial_2{}^2 A^{(2)}. \end{aligned} \tag{1}$$

Thus the magnetic field perpendicular on the average magnetic field $\mathbf{B}_\perp=(B_1, B_2)$ has a zero divergence component and a component along the direction of the gradient of $\partial_z A^{(2)}$ with non-zero divergence that is compensated by the parallel component B_z.

The two fields in Eq. (1) $A^{(1)}$, $A^{(2)}$ are considered to be stationary and homogeneous Gaussian stochastic fields, with zero average and given two-point Eulerian correlation functions $E^{(1)}$, $E^{(2)}$ defined by the statistical average

$$E^{(i)} = \left\langle A^{(i)}(0,0) A^{(i)}(x,z) \right\rangle.$$

The derivatives of the two fields are completely determined by those of $A^{(1)}$, $A^{(2)}$. They are stationary and homogeneous Gaussian stochastic fields and their two-point Eulerian correlations are obtained as derivatives of $E^{(1)}$, $E^{(2)}$. These correlation functions evidence four parameters: β_1, β_2 the amplitudes of the perpendicular magnetic field fluctuations produced by $A^{(1)}$ and respectively by $A^{(2)}$ normalized with B_0, $\lambda_\parallel$ the parallel correlation length and $\lambda_\perp$ the perpendicular correlation length. The fields $A^{(1)}$, $A^{(2)}$ are considered statistically independent.

The equations of motion of an ion with charge q and mass m in terms of the guiding center coordinate $\boldsymbol{\xi}$ and the Larmor radius $\boldsymbol{\rho}$ corresponding to the average magnetic field B_0

$$x_i = \xi_i + \rho_i, \quad \rho_i = -\varepsilon_{ij} u_j / \Omega_0, \tag{2}$$

where ε_{ij} is the antisymmetric tensor and $\Omega_0=qB_0/m$ is

$$\begin{aligned} \partial_t \rho_i &= \varepsilon_{ij} \overline{\Omega}_0 \rho_j \left[1 + b_z(\xi+\rho, z)\right] - u_z b_1(\xi+\rho, z) \\ \partial_t \xi_i &= -\varepsilon_{ij} \overline{\Omega}_0 \rho_j b_z(\xi+\rho, z) + u_z b_i(\xi+\rho, z) \\ \partial_t u_z &= \beta_1{}^2 \left[\rho_1 b_1(\xi+\rho, z) + \rho_2 b_2(\xi+\rho, z)\right] \\ \partial_t z &= \frac{u_z}{K_m}, \end{aligned} \tag{3}$$

where

$$\begin{aligned} b_z &= K_m \alpha \left(\partial_1^2 A^{(2)} + \partial_2^2 A^{(2)}\right) \\ b_i &= \varepsilon_{ij} \partial_j A^{(1)} - \alpha \partial_i \partial_z A^{(2)} \end{aligned} \tag{4}$$

Dimensionless quantities were used in Eqs. (3)-(4) with the following units: $\lambda_\perp$ for the perpendicular lengths ξ, ρ, $\lambda_\parallel$ for z, the modulus of the initial velocity of the particles u_0 for

u_z, and the time of flight over $\lambda\perp$ produced by $A^{(1)}$, $\tau = \lambda\perp/b_1u_0$ for time. The parameters of motion are the magnetic Kubo number

$$K_m = \frac{\beta_1 \lambda_{II}}{\lambda_\perp} = \frac{\tau_{II}}{\tau} \tag{5}$$

which is the ratio of the parallel decorrelation time $\tau_{II}=\lambda_{II}/u_0$ over the time of flight, β_1 the amplitude of the magnetic field fluctuations, $\alpha=\beta_2/\beta_1$ and $\overline{\Omega}_0 = \Omega_0 \tau$. Two additional parameters: the initial kinetic energy of the particle and the initial Larmor radius appear in the initial conditions for Eqs. (3)-(4).

Starting from the statistical description of the stochastic magnetic field, we determine the correlation of the Lagrangian velocity of the guiding center and the time dependent diffusion coefficient, which is the time integral of the correlation of the Lagrangian velocity.

3. The decorrelation trajectory method

The decorrelation trajectory method [2] reduces the problem of determining the statistical behavior of the stochastic trajectories to the calculation of weighted averages of some smooth, deterministic trajectories determined from the Eulerian correlation of the stochastic fields. This semi-analytical statistical approach is an approximation that satisfies the statistical consequences of the invariants of the motion. The main idea in our approach is to study the stochastic equations (3)-(4) in subensembles of realizations of the stochastic field. The whole set of realizations R is separated in subensembles (S1), which contain all realizations with the fixed values of the stochastic fields in the starting point of the trajectories **x**=0, z=0:

$$(S1): \quad A^{(1)}(0,0,0) = a^1,\, \partial_i A^{(1)}(0,0,0) = a_i^1,\, A^{(2)}(0,0,0) = a^2 \tag{6}$$

The stochastic equations (3)-(4) are studied in each subensemble (S1). The average Eulerian velocity determines an average motion in each (S1). Neglecting the fluctuations of the trajectories, the average trajectory in (S1) (*the decorrelation trajectory)* is obtained by averaging Eqs. (3)-(4) in (S1). This approximation is validated in [3]. One obtains average equations that have the same structure as the equations in each realization (3)-(4) but with the stochastic terms replaced by their subensemble averages (see [2] for details)

$$\begin{aligned} B_z &= \langle b_z \rangle_{S1} = K_m \alpha a^2 \left(\partial_1^2 + \partial_2^2\right) E^{(2)} \\ B_i &= \langle b_i \rangle_{S1} = \varepsilon_{ij} \partial_j \left(a^1 E^{(1)} - a_k^1 \partial_k E^{(1)}\right) - \alpha a^2 \partial_i \partial_z E^{(2)} \end{aligned} \tag{7}$$

The time dependent diffusion coefficient is obtained by summing the contribution of each subensemble (S1) weighted by the probability that a realization belongs to the subensemble.

4. Results and conclusions

The topology of the decorrelation trajectories is directly correlated with the nonlinear effects in stochastic transport. They show in this case that there are two nonlinear effects that produce trajectory trapping. The first appears for the motion in the plane perpendicular to the average magnetic field and consists in helicoidal segments of the magnetic lines, which form localized magnetic structures (magnetic islands). This trapping process is specific to the 2-dimensional stochastic magnetic fields perpendicular to a large $\mathbf{B}_0$ but it could be observed also in the 3-dimensional case. The second trapping effect appears in particle motion along magnetic lines and is due to the formation of stochastic magnetic mirrors. Localized structures can appear due to this effect of trapping (trajectory clusters).

The interaction of the two types of trapping is rather complex and determines either the release of trajectories or a synergetic amplification or even chaotic behavior. Their effects on the diffusion coefficient depends on the statistical weight of each type of decorrelation trajectories. A rich class of anomalous diffusion regimes are identified, in agreement with the results of numerical simulations [5]. The magnetic Kubo number provides a measure of the nonlinear effects (as in the 2-dimensional case) but other parameters can change the diffusion regime. For instance, the dependence of D on K_m is step-like for Ω_0 above a limit value and irregular at smaller values. The mirror capture appears for large values of K_m and leads to the increase of the perpendicular trapping effect and thus of the perpendicular diffusion coefficient.

In conclusion, we have shown that the nonlinear effects are complex and strong in 3-dimensional magnetic fields. A physical image was obtained by analyzing the topology of the decorrelation trajectories.

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