A kinetic study of dust grain screening based on the numerical solution of the Vlasov-Bhatnagar-Gross-Krook equations

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Introduction. A study of dust grain screening is of great importance for dusty plasma theory, since the grain charge, together with the electric potential distribution around the grain, determines the intergrain forces, which are responsible for a variety of collective phenomena in dusty plasmas [1]. In recent years considerable attention has been given to the effect of plasma collisions (mainly ion-neutral collisions) on the considered process [2]. In the present work, this problem is studied using an approach based on the numerical solution of the Vlasov-Bhatnagar-Gross-Krook (Vlasov-BGK) kinetic equations. A detailed description of governing equations and numerical procedure can be found in our previous works [3, 4]. In this short paper we briefly outline only the main points of the approach.

Formulation of the problem. Let us consider a spherical dust grain of radius a immersed in a weakly ionized argon plasma consisting of electrons (e), atoms (a) and singly ionized ions (i). The plasma background is characterized by temperatures T_α^0 and concentrations n_α^0, where $\alpha = \mathrm{e}, \mathrm{i}, \mathrm{a}$ denotes the type of a plasma particle. The gas of neutral atoms is assumed to be in an undisturbed stationary state, and the dynamics of ions and electrons is governed by the Vlasov-BGK equations, which are considered in the spherically symmetric form. The kinetic equations are supplemented by the equation for the self-consistent electric field and grain charging equation. The collisions in plasma are described using the model collision integrals proposed in [5] on the basis of the consistent BGK-type model for gas mixtures. These integrals satisfy all fundamental properties of the Boltzmann collision integrals, such as conservation laws, positivity, correct exchange coefficients, and entropy inequality. This is in contrast to other similar works, in which a more simple form of the model collision integrals is used. The Vlasov-BGK system of equations are solved numerically on parallel processors by means of the high-order finite-volume method developed in [3, 4].

Results and discussions. In this section, we summarize the results of numerical computations demonstrating the effect of ion-neutral collisions on the grain charging and screening processes. Let us consider first the case of isothermal plasma, which is assumed to be in ionization (Saha) equilibrium at some temperature T_0 and pressure P_0. The temperature T_0 in our study varies from 0.4 to 0.5 eV and the pressure P_0 varies in a wide range from 10^{-4} to 5 atm.

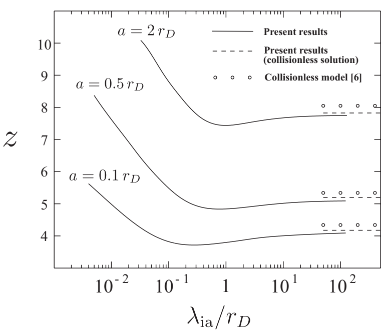

Figure 1: The normalized grain charge z versus the ion collisionality index λ_{ia}/r_D.

Under these conditions, the Debye length r_D, that is given by $r_D^{-2} = 4\pi e^2 \left(n_e^0/kT_e^0 + n_i^0/kT_i^0\right)$, is approximately equal to $5-7\,\mu$m and can be comparable to the typical grain sizes used in laboratory investigations of complex plasma. Thus, the grain radius a was chosen to be equal to $0.1\,r_D$, $0.5\,r_D$ and $2\,r_D$. In Fig. 1, we show the dependence of the normalized grain charge $z = Qe/akT_e^0$ on the ion collisionality index λ_{ia}/r_D for different grain sizes, where λ_{ia} is the mean free path for ion-neutral collisions and Q is the total grain charge. As one can see in Fig. 1, the dependence of the normalized charge on the ion collisionality is nonmonotonic and has a minimum at some value of the collisionality index. The physical reasons for the occurrence of this minimum were already discussed in [2]. We also observe that the magnitude of the grain charge increases with the grain radius. This result is in agreement with [6, 7].

In Fig. 2, we show the distributions of the normalized electric potential $e\varphi/kT_0$ for different collisional regimes in the case of $a = 0.5\,r_D$. For two limiting cases of the collisionless and strongly collisional background, our results are found to be in good agreement with those computed using the commonly known approaches (the collisionless model [6] and drift-diffusion model [7]). As it is known, in the limit of strongly collisional plasma, the electric potential has a Coulomb-like asymptote [7]. We can observe that for intermediate collisional regimes, the electric potential also has a Coulomb-type asymptotical behavior, but the residual charge

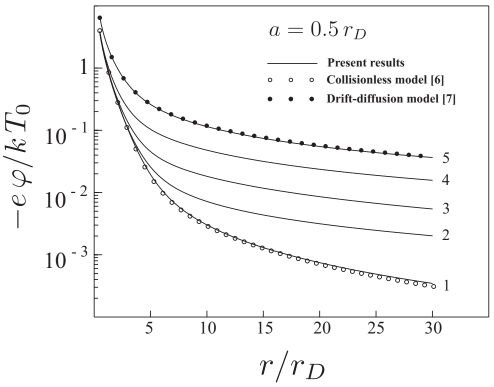

Figure 2: Distributions of the normalized electric potential in the case of $a = 0.5\,r_D$ for different collisional regimes: collisionless (1), $\lambda_{ia}/r_D = 3.28$ (2), 1.16 (3), 0.38 (4), 0.012 (5);.

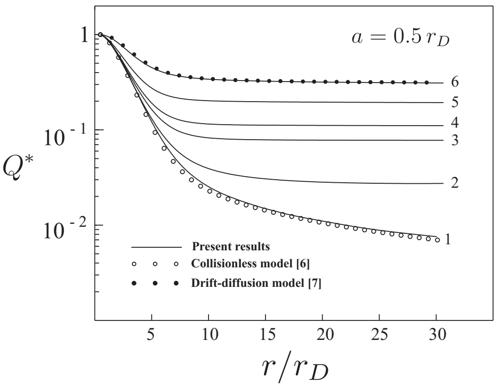

Figure 3: Distributions of the effective charge in the case of $a = 0.5\,r_D$ for different collisional regimes: collisionless (1), $\lambda_{ia}/r_D = 1.16$ (2), 0.79 (3), 0.38 (4), 0.03 (5), 0.01 (6)

is smaller in magnitude. In order to demonstrate this statement clearly, we give the relative charge distributions for different collisional regimes in Fig 3. The relative charge is defined as $Q^* = Q_r/Q$, where Q_r is the total charge residing within a sphere of radius r. As is seen from Fig. 3, the relative charge becomes constant at large distances (except for collisionless solution). According to the Ostrogradsky-Gauss theorem, it means that the electric field has the Coulomb-type asymptotical behavior.

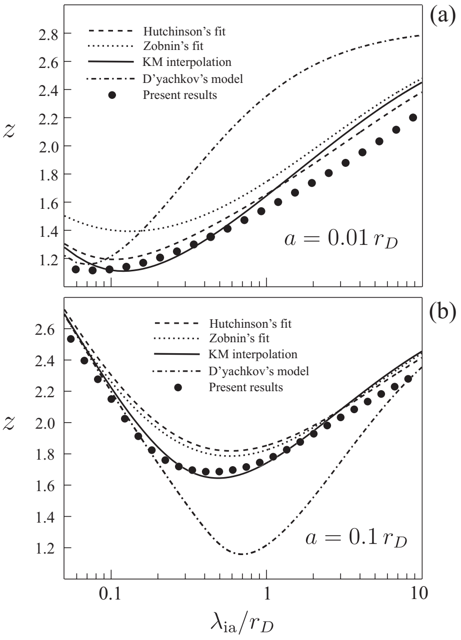

Figure 4: The normalized grain charge z versus $\lambda_{\mathrm{ia}}/r_D$. The results are presented for two grain sizes: $a = 0.01\,r_D$ (a) and $a = 0.1\,r_D$(b).

Let us consider further the case of non-isothermal discharge plasma. The following representative parameters were used in our numerical computations: $T_{\mathrm{i}}^0 = T_{\mathrm{a}}^0 = 0.03\,\mathrm{eV}$; $T_{\mathrm{e}}^0 = 30\,T_{\mathrm{i}}^0$; $n_{\mathrm{i}}^0 = n_{\mathrm{e}}^0 = 10^{14}\,\mathrm{m}^{-3}$; and plasma pressure P_0 ranges from 1 to 200 Pa. Under these conditions, the Debye length r_D is approximately equal to $100\,\mu\mathrm{m}$ and the relation $a \ll r_D$ holds for typical grain sizes. Thus, the grain radius a was chosen to be equal to $0.1\,r_D$ and $0.01\,r_D$. In Fig. 4, we demonstrate the dependence of the normalized grain charge z on the ion collisionality index $\lambda_{\mathrm{ia}}/r_D$. In order to compare our results with those obtained using other approaches, we also show in Fig. 4 the curves corresponding to different models reviewed in [2] (the Hutchinson's fit, Zobnin's fit, KM interpolation and D'yachkov's model). One can observe from Fig. 4 that our results are in good agreement with those based on the Hutchinson's fit, Zobnin's fit and KM interpolation formula. The results obtained using the D'yachkov's model deviate noticeably from those based on other approaches. It can be explained by the fact that this model is derived under an assumption of $\lambda_{\mathrm{ia}} \ll r_D$, which is usually not satisfied in the weakly collisional regime.

Let us also discuss the influence of ion-neutral collisions on the grain screening process in the considered case. This problem has been previously analyzed in [8] on the basis of a simple linear kinetic model with point-sink approximation. The expression for the elecric potential presented in [8] is derived under an assumption of $\lambda_{\mathrm{ia}} \gg r_D$, but it also gives correct results in the strongly collisional limit and can be used, at least formally, to calculate the electric potential for arbitrary values of the ion collisionality index. In Fig. 5 we demonstrate the results of comparison between our numerical calculations and those obtained using the model of [8]. We show the distributions of the normalized electric field $\hat{E} = Er_D e/kT_e^0$ in the case of $a = 0.01 r_D$ and $\lambda_{ia} = 0.3 r_D$. Our computations show that the for the most part there is good agreement between our results and those of [8] and the differences arise mainly in the intermediate collisional regime.

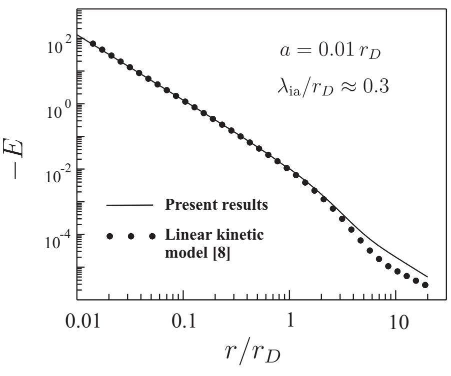

Figure 5: Distributions of the normalized electric field $\hat{E}$ for $a = 0.01\, r_D$, $\lambda_{ia} = 0.3 r_D$

Conclusions. To conclude, in the present work, the effect of ion-neutral collisions on the dust grain charging and screening processes is studied on the basis of the Vlasov-BGK equations. The kinetic equations are solved numerically on parallel processors using the high-order finite-volume method developed in our previous works [3, 4]. The cases of both thermal and nonisothermal discharge plasma are considered. It is shown that there is a good agreement between our numerical results and those obtained using the known nonlinear collisionless model [6] and drift-diffusion model [7], which represent the two limiting cases of collisionless and strongly collisional background, respectively. For the transitional regime, it was established that the electric potential also has the Coulomb-type asymptotical behavior related to the presence of the unscreened residual grain charge. A comparison between our results and those obtained using different analytical models describing the dependence of the grain charge on the ion collisionality [2] is presented. In addition, the applicability of a simple linear kinetic model presented in [8] is examined. It is shown that the model of [8] gives a reasonable approximation for the electric potential in a wide range of collisional regimes.

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