Magnetic phase transitions in plasmas and transport barriers

Emilia R. Solano[1], Richard D. Hazeltine[2]

[1] *Asociación EURATOM-CIEMAT para Fusión, Madrid, Spain*

[2]*Institute for Fusion Studies, Austin ,USA*

The prevailing paradigm of the L-H transitions is that the H-mode is achieved when velocity $\vec{E}\times\vec{B}$ shear is sufficient to stabilise plasma electrostatic fluctuations responsible for anomalously high transport. Nevertheless the transition trigger mechanism remains unknown. Here we present a study of the dynamics of *magnetic phase transitions* in plasmas, and show how they influence plasma confinement, leading to transport barrier formation. The mechanism we propose is very simple: low pressure plasmas are paramagnetic and attract low pressure plasmas, becoming more paramagnetic. Conversely, high pressure plasmas are diamagnetic and attract diamagnets. I.e., the magnetisation state of the background plasma, para or dia, determines the motion of magnetised pressure blobs and affects profile evolution.

In tokamaks the dominant extrinsic magnetic field is toroidal. Poloidal currents flowing in the plasma and enclosing the magnetic axis can be diamagnetic, decreasing this externally applied toroidal field, or paramagnetic, increasing it. The boundary between para and diamagnetic radial regions in the bulk plasma has zero magnetisation current: $j_\theta = 0$.

Consider any plasma cylinder, with an externally imposed magnetic field B_z and a pressure profile peak at the centre, as depicted in red at the top of Fig. 1, left. Assume equilibrium

$$\mathbf{F} = n\, m\frac{d\mathbf{v}}{dt} = -\nabla p + \mathbf{j}\times\mathbf{B} = 0$$

If the pressure profile is peaked (red on top left of Fig.1) the perpendicular current is diamagnetic. Conversely, a hollow pressure profile (blue in Fig. 1 bottom left) would be surrounded by a perpendicular paramagnetic current. If there is a longitudinal free current flowing in the plasma B becomes helical. A current parallel to B such that $\mathbf{j}_{\|}\cdot\mathbf{j}_z > 0$ (like the bootstrap current in tokamaks) has a paramagnetic $\mathbf{j}_{\|\theta}$ (Fig. 1, right).

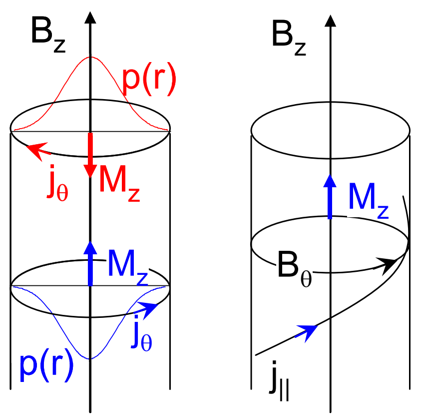

Fig. 1: *Left*: sketch of plasma magnetization associated with pressure gradients in cylindrical plasma. *Right*: magnetisation associated with co-parallel current.

Consider a background plasma in equilibrium, with pressure p_0, confined by a magnetic field **B**. Consider next a pressure blob (excess or defect of pressure, elongated along a field line [1,2]). At the blob location the total pressure is $p = p_0 + \tilde{p}$. The blob's centre is given by local extrema of p, and its radius is by the nearest location where $\nabla(p_0 + \tilde{p}) = 0$.

The perpendicular current density and corresponding blob magnetisation are [3]

$$\tilde{\mathbf{j}}_\perp = \frac{\mathbf{b}\times\nabla\tilde{p}(\rho)}{\bar{B}} = \nabla\times\tilde{\mathbf{M}} \qquad \text{and} \qquad \tilde{\mathbf{M}} = \frac{1}{\lambda_\parallel}\int_0^\rho \frac{\mathbf{b}}{B}\frac{\partial\tilde{p}(\rho')}{\partial\rho'}\lambda_\parallel\, d\rho' \approx -\frac{\tilde{p}}{B}\mathbf{b}$$

The radial force balance equation describing the trajectories of magnetised blobs under the influence of the background plasma magnetization is obtained by Taylor expansion of the magnetic field from the centre of mass of the tube, integrated over the blob volume [4]:

$$m_{blob}\frac{d\mathbf{v}_{blob}}{dt} = \int_V\left(-\nabla\tilde{p} + \tilde{\mathbf{j}}\times\mathbf{B}\right)dV =$$

$$= \int_{\tilde{V}}\left(\left(-\nabla\tilde{p} + \tilde{\mathbf{j}}_\perp\times\mathbf{B}_0\right) + \tilde{\mathbf{j}}_\perp\times(\boldsymbol{\rho}.\nabla\mathbf{B})\right)dV$$

$$\bar{\rho}_{m,blob}\frac{d\mathbf{v}_{blob}}{dt} = \frac{1}{\tilde{V}}\int_{\tilde{V}}\left(\nabla\left(\tilde{\mathbf{M}}\cdot\mathbf{B}\right)\right)dV$$

We concentrate on toroidal magnetisation $\tilde{\mathbf{M}}_\zeta$ and assume $\tilde{M}_\zeta\nabla B_\zeta >> B_\zeta\nabla\tilde{M}_\zeta$. Blob dynamics is then given by:

$$\bar{\rho}_{m,blob}\frac{dv_r}{dt} = \tilde{M}_\zeta\frac{d\bar{B}_\zeta}{dr} = -\mu_0\tilde{M}_\zeta\bar{j}_\theta \qquad (1)$$

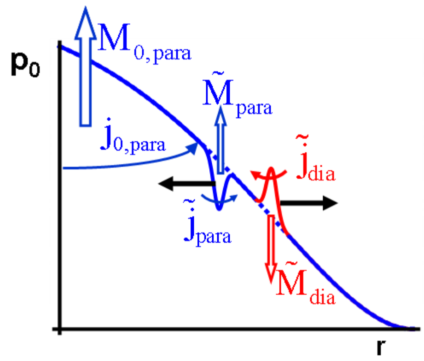

Fig. 2 Paramagnetic pressure profile with high (red) and low (blue) pressure blobs. Paramagnetic plasmas attract paramagnetic blobs, repel diamagnetic blobs (& vice-versa).

The overbar indicates an average of the background quantity over the blob volume, $\bar{\rho}_{m,blob}$ is the average blob mass density. Equation (1) shows that the toroidal magnetic field of the background plasma provides an anti-potential for motion of local magnetized blobs. In a paramagnetic region $d\bar{B}_\zeta/dr < 0$ and a paramagnetic blob is driven inwardly, up the hill of the paramagnetic field. This means that low pressure fluctuations move towards paramagnetic plasma regions, while high pressure fluctuations escape from them: a paramagnetic plasma has "low confinement", L-mode. Conversely, high pressure blobs move towards diamagnetic regions: high confinement, H-mode.

Note that the mechanism of phase separation looks like a growing instability in paramagnetic regions: as a cold blob moves inwards towards higher pressure its "amplitude" (the difference between $\tilde{p}$ in the blob centre and the ambient pressure) increases. A hot blob moving outwards also would appear to grow as it moves towards lower pressure regions. On the other hand, a hot blob moves inwards (up the pressure gradient) in a diamagnetic region, until it eventually encounters a matching background pressure. The blob then merges with the background pressure (adding to its gradient) and ceases to exist. Through this process the "amplitude" of both hot and cold blob decreases, so we would classify this behaviour as stable, self-limiting. Still it drives energy fluxes and reinforces magnetic phase differentiation.

Note that in general an outer diamagnetic plasma layer surrounding a paramagnetic one is energetically favourable, as $B_t^2 / 2\mu_0$ is minimised.

How does blob migration affect profile evolution? In the evolution equations

$$\frac{3}{2}\frac{\partial p}{\partial t} + \nabla.\mathbf{Q} = H$$

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla\times(\mathbf{v}\times\mathbf{B}) + \nabla\times\eta(\mathbf{j}-\mathbf{j}_{ni}) = 0$$

the contribution of the magnetization force (subscript M) to evolution of surface averaged quantities can be estimated from the convective flux of blobs as

$$\left\langle \frac{3}{2}\frac{\partial p}{\partial t} \right\rangle_M = -\nabla.\langle \tilde{p}\, \mathbf{v}_r \rangle$$

$$\left\langle \frac{\partial \mathbf{B}_z}{\partial t} \right\rangle_M = \nabla\times\langle (v_r B_z \boldsymbol{\theta}) \rangle$$

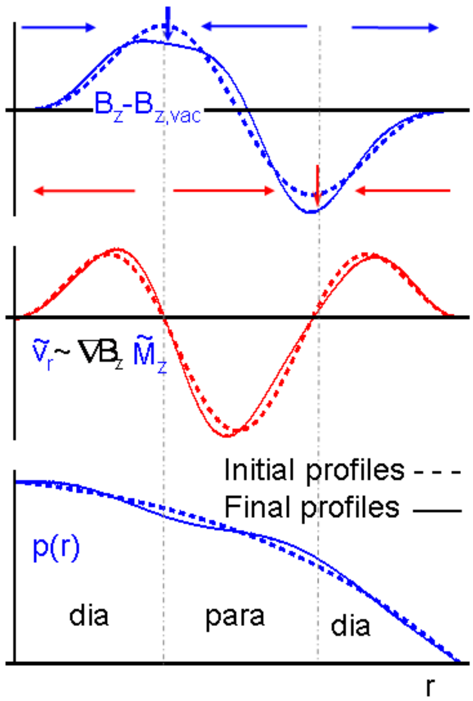

Fig. 3: Plasma evolution cartoon (initial states in dashed lines). a) toroidal field in magnetized plasma, b) velocity of paramagnetic blobs and c) radial pressure profile.

With initial model profiles (Fig. 3 dashed profiles), we evaluate the effect of a radial velocity proportional to (1). The velocity of paramagnetic tubes is shown in Fig. 3.b, and represented by horizontal blue arrows in Fig. 3a. Paramagnetic plasma elements seek high magnetic field, while diamagnetic elements take their heat to the magnetic well (red arrows). The changed magnetisation and pressure are illustrated with solid lines in 3.a, 3.c. We see that the magnetisation force naturally creates a pedestal structure by adding heat to the magnetic well, and removing it from the magnetic hill: pressure is flattened at the pedestal top, steeper outwards, forming a "transport barrier". Further, the tendency is for the diamagnetic well to become narrower and deeper, while the hill becomes broader: barrier steepening.

In a toroidal system short blobs do not sample the flux surface averaged magnetisation, so they can respond to local B_ζ gradients. The 1/R variation of the external B_ζ field would drive cold blobs into the plasma at the LFS, and hot blobs out, but the effect is reversed in the HFS. Global phase transition can only take place when plasma magnetisation overcomes the 1/R dependency of the field, or when the blobs are sufficiently elongated to sample an average B_ζ.

The averaged magnetisation force is a radial force that acts equally on both ions and electrons, and therefore can give rise to interchange instabilities [5], the plasma equivalent of the Raleigh-Taylor instability. The transport mechanism associated with the magnetisation force can be most easily understood as a magnetisation interchange: blobs drift under the influence of magnetic field gradients, interchanging plasma thermal pressure and toroidal magnetic field pressure. In our model the plasma background magnetisation replaces gravity, with the peculiarity that it can have either sign. Interchange linear growth rate can be estimated as $\gamma = \sqrt{g\kappa_\perp} \simeq \sqrt{\frac{\tilde{M}_\zeta}{\bar{\rho}_{m,blob}} \frac{d\bar{B}_\zeta}{dr} \frac{1}{\lambda_\rho}} \simeq \sqrt{-\frac{1}{\bar{\rho}_{m,blob}} \frac{\tilde{p}}{\bar{B}} \frac{d\bar{B}_\zeta}{dr} \frac{1}{\lambda_\rho}}$, assuming $\kappa_\perp \sim 1/\lambda_\rho$. Inspection of the signs in this equation show positive growth rates for mixed states. But as we mentioned earlier, growth rates of linear instabilities do not tell the whole story. Seed pressure fluctuations may or may not originate in magnetization interchange instabilities. If they do not, the creation rate of blobs might control transport rates, but the magnetisation interchange mechanism would still drive a magnetic phase transition and a confinement transition.

The model postulates that a necessary condition for the L to H transition to take place is that the plasma background must have a magnetisation boundary, $\nabla p = \vec{j}_\zeta \times \vec{B}_\theta$ near the edge. In a steady L-mode plasma the toroidal current density can be estimated from loop voltage measurements and plasma resistivity η. We therefore propose a testable necessary condition for an H-mode transition is that at the plasma edge $|\nabla p| \geq |j_\zeta B_\theta| = |V_{loop} B_\theta / 2\pi R\eta|$

In summary: from a first principles model of plasma magnetisation and its effect on pressure and toroidal field profile evolution we show that magnetic phase transitions might explain confinement transitions and pedestal formation [6].

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