Magnetic Field Effects on Plasma Plumes

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Space plasma propulsion systems currently being developed require strong guiding magnetic fields known as magnetic nozzles to control plasma flow and produce thrust. Among these propulsion methods are the VAriable Specifc Impulse Magnetoplasma Rocket (VASIMR)[1], magnetoplasmadynamic thrusters (MPDs), and helicon thrusters. Magnetic nozzles are functionally similar to de Laval nozzles, but are inherently more complex systems due to the plasmadynamics resulting from the magnetic field effects on the plasma plume. Here, we perform a preliminary study of numerical magnetic nozzle experiments.

The crucial physical phenomenon of magnetic nozzles are thrust production and plasma detachment. The physics of thrust production encompasses conversion of magnetoplasma energy into directed kinetic energy as well as the mechanisms for transferring momentum to the spacecraft. The physics of plasma detachment governs the separation of plasma from the spacecraft and must be understood to optimize nozzle design for maximum efficiency. The mechanisms for inducing efficient detachment are an active research topic. The goal of this research is to perform numerical experiments in the magnetohydrodynamic (MHD) regime observing which physical phenomena occur and optimizing magnetic nozzle design accordingly.

To perform numerical experiments a novel, hybrid kinetic theory and single fluid MHD solver known as the Magneto-Gas Kinetic Method (MGKM) was developed[2]. The solver is comprised of a "fluid" portion that finds solutions to the Navier Stokes equations through the Gas Kinetic Method (GKM)[3] and "magnetic" portion that incorporates MHD physics through source terms to the conserved fluid variables. A generalized Ohm's law and Maxwell's equations are used to close the system of equations and self-consistently calculate the induced magnetic field. The system of equations is shown below with the "fluid" portion on the left side and the "magnetic" portion on the right side of the conservation equations.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = 0 \tag{1}$$

$$\frac{\partial (\rho \mathbf{U})}{\partial t} + \nabla \cdot \left[\rho \mathbf{U}\mathbf{U} + \underline{\underline{\tau}} + \underline{\underline{I}} p\right] = \mathbf{J} \times \mathbf{B} \tag{2}$$

$$\frac{\partial e}{\partial t}+\nabla\cdot\left[\mathbf{U}(e+p)-k\nabla T+\mathbf{U}\cdot\underline{\underline{\tau}}\right]=\mathbf{J}\cdot\mathbf{E} \tag{3}$$

$$\mathbf{J}=\frac{\nabla\times\mathbf{B}}{\mu_0} \tag{4}$$

$$\frac{\partial\mathbf{B}}{\partial t}=-\nabla\times\mathbf{E} \tag{5}$$

In the above equations $\underline{\underline{\tau}}$ is the dissipative stress tensor and e is the hydrodynamic energy defined as $e=\rho U^2/2+p/(\gamma-1)$.

A parametric study of the generalized Ohm's law (6) was performed to determine the type of MHD solver necessary to capture the relevant magnetic nozzle physics.

$$\mathbf{E}=-\mathbf{U}\times\mathbf{B}+\frac{1}{n_e e}\mathbf{J}\times\mathbf{B}-\frac{1}{n_e e}\nabla(n_e kT_e)+\eta\mathbf{J} \tag{6}$$

The terms on the right side of (6) will be referred to as the convective, Hall, electron pressure, and resistive terms respectively. To maintain a single fluid model the plasma is assumed to be quasi-neutral ($n_i=n_e$) and single temperature ($T_i=T_e$). Table 1 shows the results of the parametric analysis. Note that these ratios are calculated primarily using data near the nozzle throat where the magnetic field is strongest. In this table ω, ω_{ce}, ω_{ci}, ν_{ei}, and Rm refer to the characteristic flow frequency, electron cyclotron frequency, ion cyclotron frequency, electron-ion collision frequency, and the magnetic Reynolds number respectively.

		Experiments			
Ratio of Terms	**Equation**	**VASIMR-Helicon**	**VASIMR-ICH**	**HPH**[8]	**DDEX** [9]
MHD Assumption	ω_{ce}/ω	$\approx 10^6$	$\approx 10^5$	$\approx 10^4$	$\approx 10^6$
Hall/Convective	ω/ω_{ci}	$\approx 10^{-1}$	$\approx 10^{-1}$	$\approx 10^0$	$\approx 10^{-3}$
Resistive/Convective	$1/Rm$	$\approx 10^{-3}$	$\approx 10^{-4}$	$\approx 10^{-2}$	$\approx 10^{-3}$
Hall/Resistive	ω_{ce}/ν_{ei}	$\approx 10^2-10^3$	$\approx 10^2-10^3$	$\approx 10^2$	$\approx 10^2$
Hall/Electron Pressure	p_B/p_e	$\approx 10^2-10^3$	$\approx 10^1-10^2$	$\approx 10^0$	$\approx 10^1$

Table 1: Parametric Analysis

The parametric analysis shows that the primary assumption for simplifying the electron equation of motion into the generalized Ohm's law is satisfied in all systems analyzed. The analysis also shows that Hall term effects must be included. Furthermore, Hall effects may become greater downstream due to the decrease in applied field strength. The resistive term appears to be small compared to the convective and Hall terms, but is incorporated to eliminate numerical stiffness, include MHD turbulence effects, and account for the contribution of cross field diffusion to detachment. Additionally, experiments have shown cross field diffusion to be much greater than expected due to anomalous resistivity, which may be incorporated in the future[4]. The electron pressure term is found to be important in some cases, but not others, therefore it is included for completeness of the model. In summary, we found that the generalized Ohm's law is necessary to capture the relevant physics of magnetic nozzles and developed MGKM accordingly. The new solver was validated through the study of MHD shock tube, Hartmann, and Couette flows.

Characteristic	Equation	Approximate Values (m/s)
Fluid	U	$\approx 2 \cdot 10^4$
Alfvén (V_A)	$B/\sqrt{\rho\mu_0}$	$\approx 6 \cdot 10^4$
Magnetosonic (V_M)	$\sqrt{V_A^2 + C_s^2}$	$\approx 6 \cdot 10^4$
Hall Velocity	J/ne	$-$
Hall Drift Velocity	$\frac{B_0}{n^2 e \mu_0} \frac{\partial n}{\partial x}$	$-$
Whistler Wave (V_W)	kV_A^2/ω_{ci}	$\approx 2 \cdot 10^5/(\Delta x)$

Table 2: MHD characteristic velocities of VX-200 with Helicon only operation

The computational challenges associated with modeling magnetic nozzle plasma flows using a generalized Ohm's law MHD solver are numerous and non-trivial. The range of characteristic velocities of Hall MHD for a numerical test case of the 200 kW VASIMR Experiment(VX-200) are shown in Table 2. The whistler wave characteristic appears due to the incorporation of the Hall term in the generalized Ohm's law and is the most restrictive in determining time step size. Typical grid sizes on the order $10^{-2} - 10^{-1}$ (m) result in time steps which are two to three orders of magnitude smaller than those without the Hall term. This results in significant increases in computational effort. An inherent difficulty in solving the MHD equations numerically is the lack of an explicit way to satisfy Maxwell's equation requiring $\nabla \cdot \mathbf{B} = 0$. This condition is analytically satisfied for all time if is initially satisfied, but numerical errors violating this condition have the potential for unmitigated growth due to the lack of an equation enforcing it. We have a method implemented in our solver which should alleviate this numerical error, but have found that strong shocks cause this method to fail. A number of numerical methods which address this error will be considered in the future [5]. Expansion of the fluid into a vacuum is also problematic due to the formation of steep gradients and shocks which may lead the numerical method to produce negative densities and temperatures. This problem may be resolved in the future through the use of logarithmic variables [6]or a multi-fluid GKM solver. Cartesian grids also present a numerical challenge when modeling the inherent cylindrical symmetry of a plasma jet with the grid affecting results unless very fine meshes are used. Finally, the currently first-order time-accurate MGKM forces the time steps to be prohibitively small, but this can obviously be mitigated by using a higher-order time-integration method.

In conclusion, we have shown that it is necessary to include the full generalized Ohm's law to capture the plasmadynamics in magnetic nozzles of current experimental setups. We identified the numerical challenges associated with using this type of model and have identified methods that can be used to overcome them. This research is funded by a NASA Space Technology Research Fellowship grant number NNX11AM98H and will be continued by the primary author at the University of Texas at Austin. The development of GKM and MGKM will continue at Texas A& M University where it will be used for direct numerical simulation of turbulence [7] as well as the study of MHD plasma flows. The authors would like to thank NASA, Texas A&M University, and Ad Astra Rocket Company for their support in this research.

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