PIC Simulations of the Seeded Self-Modulation of a Long Proton Bunch in Plasma Density Gradients

P. I. Morales Guzman[1], J. Vieira[2], P. Muggli[1], AWAKE Collaboration

[1] *Max Planck Institut für Physik, Munich, Germany*

[2] *GoLP/Instituto de Plasmas e Fusão Nuclear, IST, Lisbon, Portugal*

Introduction

In a plasma wakefield accelerator, a long proton bunch can be used as a driver when it undergoes the process of self-modulation (SM) [1], as shown in the AWAKE Experiment [2, 3]. In the SM, the proton bunch transforms into a series of microbunches shorter than and approximately separated by the plasma wavelength λ_{pe}, that resonantly drives wakefields. To produce wakefields with a large amplitude over distances interesting for electron acceleration to very high energies, the microbunches should ideally be positioned in the focusing and decelerating phase.

During SM, the phase velocity of the wakefields v_{ph} is slower than the bunch velocity v_b [4, 5] as:

$$v_{ph} = v_b \left(1 - \frac{1}{2}\left(\frac{\xi}{z}\right)^{1/3}\left(\frac{n_b m_e}{2 n_e m_p \gamma}\right)^{1/3}\right), \tag{1}$$

where ξ is the position along the bunch, z is the position along the plasma, n_b and n_e are the bunch and plasma densities, γ is the relativistic factor of the bunch, and m_p and m_e are the proton and electron masses.

Table 1: Simulation parameters.

Plasma and window parameters	**Physical value**	**Norm.value**
Initial plasma density	$1.81 \times 10^{14}\,\text{cm}^{-3}$	$1\ n_{e0}$
Plasma radius	0.15 cm	$3.8\ c/\omega_p$
Plasma length	10.2 m	$25850\ c/\omega_p$
Simulation window length	21 cm	$532\ c/\omega_p$
Simulation window width	0.158 cm	$4\ c/\omega_p$
Longitudinal resolution	5.9 µm	$0.015\ c/\omega_p$
Transverse resolution	4 µm	$0.01\ c/\omega_p$
Time step	9.2 fs	$0.007\ 1/\omega_p$
Particles per cell	–	3×3
Bunch parameters		
RMS radius (σ_{r0})	200 µm	$0.51\ c/\omega_p$
RMS length (σ_z)	6.9 cm	$176.85\ c/\omega_p$
Norm. emit. (ε_N) / RMS $p_\perp$ spread	3.6 mm mrad	$0.018\ m_p c$
Seed position (ahead of bunch center)	3.81 cm	$96.4\ 1/\omega_p$
Relativistic factor (γ)	426.44	426.44
Relative energy spread	0.035 %	0.035 %
Population/peak density	3×10^{11} protons	$0.038\ n_0$
Particles per cell	–	2×2

The further evolution of the microbunch train causes the microbunches to be partially in the defocusing phase. These protons are eventually pushed away from the axis and leave the wakefields. Some of the protons may also fall into the accelerating phase at some point along the plasma, taking energy away from the wakefields. Both effects decrease the amplitude of the wakefields.

Using a plasma with a linear density gradient g has an effect on λ_{pe} and therefore on the phase of the wakefields with respect to the protons. A negative gradient makes λ_{pe} longer along the plasma, enhancing the difference in velocities. A positive gradient has the opposite effect and can be used to mitigate the deleterious effects of the slow wakefields on the amplitude of the wakefields.

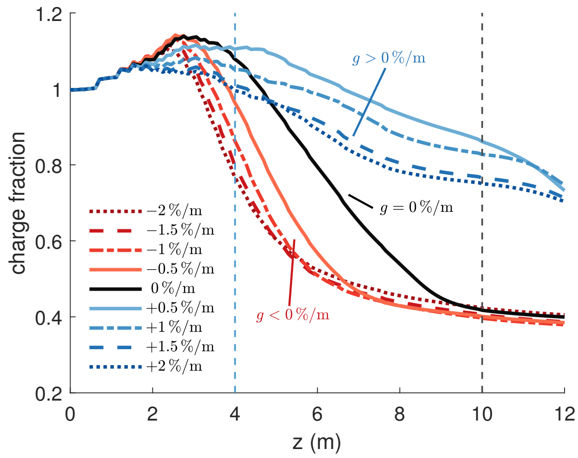

Figure 1: *Proton bunch charge fraction within one σ_r radius of the incoming proton bunch along the plasma.*

We perform simulations in 2D axisymmetric geometry using Osiris 4.4.4 [7] with parameters similar to those of the experiment [6], which are listed in Table 1. We use a step in the proton bunch density profile to seed the wakefields.

Evolution of the charge fraction in the microbunch train

In Fig. 1, we calculate the charge fraction in the microbunch train by considering the charge within one rms width of the bunch $\sigma_r(z) = \sigma_0\left(1+\frac{z^2\varepsilon_N^2}{\gamma^2\sigma_0^4}\right)$, where σ_0 is the initial rms radius of the bunch, and dividing by the charge in that region of the unmodulated bunch. Finally we add $\approx 11\,\%$, which is the charge fraction ahead of the seeding position, that is present in the experiment, but not in simulations.

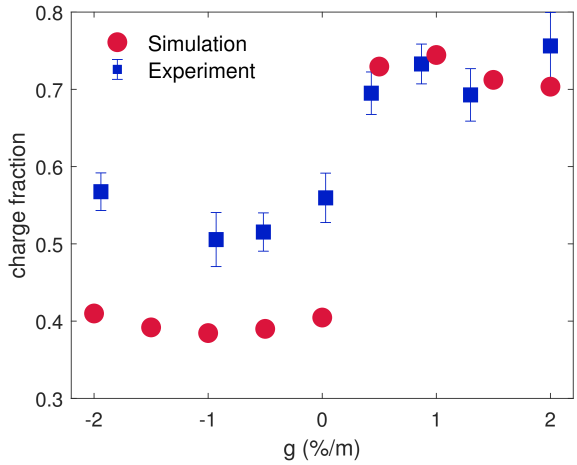

Figure 2: *Proton bunch charge fraction within one σ_r radius of the incoming proton bunch after 10 m of propagation in plasma and 2 m in vacuum, from the experiment (squares, from Ref. [6]) and from simulations (circles).*

Initially, the charge fraction is larger than 1 as the focusing fields (transverse wakefields and adiabatic plasma response) concentrate the charge on axis. After 3 m, there is a significant separation between charge fractions for $g > 0$ and $g < 0$, and $g = 0\,\%/\mathrm{m}$ remains in between. The charge fraction at $z = 12\,\mathrm{m}$, the equivalent location of the screen of the experiment, corresponds to the values in Fig. 2, where there is a good general agreement with the experimental values [6]. Simulation results indicate that the charge fraction value is mainly the result to the dephasing of the fields with respect to the microbunch train: large for $g < 0$, small for $g > 0$.

Waterfall plots of the longitudinal wakefields

In Fig. 3, we show waterfall plots of the longitudinal wakefields on axis for three positions along the bunch. These are built by taking a lineout of the wakefields on axis and stacking them upwards from the plasma start ($z = 0$ m) to its end ($z = 10$ m). By following the zero-crossing of those fields, one can have a handle of the phase of the wakefields. In those plots we also show the approximate position of one typical microbunch by following the peak in its density profile starting at $z \approx 1$ m. The opacity of the line following that peak corresponds to the value of the peak, normalized to its maximum value. Therefore, when the peak has a low value, the line becomes transparent. On these figures the velocity of the protons ($\approx c$) is a vertical line. Lines with negative slope ($dz/d\xi < 0$) have a slower velocity.

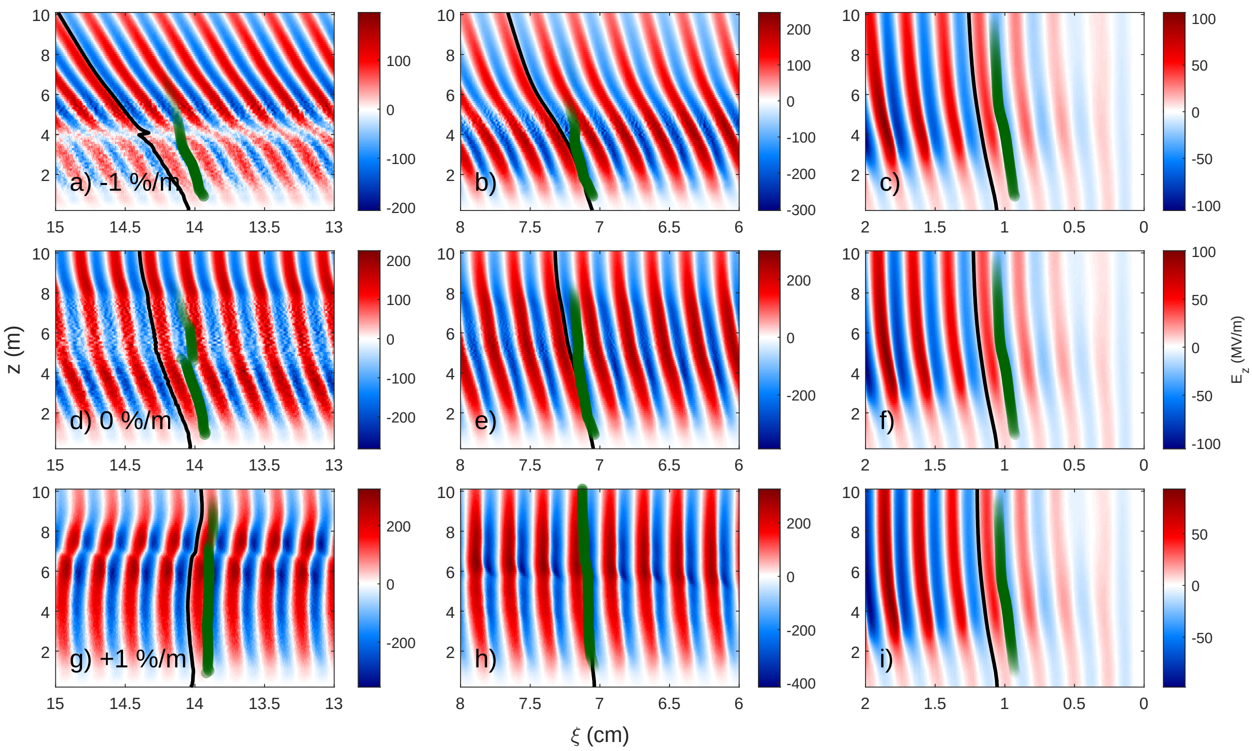

Figure 3: *Waterfall plots of the longitudinal wakefields on axis around three positions along the bunch: $\xi_0 = 1$ (c, f, i), 7 (b, e, h), and 14 cm (a, d, g) for three g: $g = -1$ %/m (a-c), $g = 0$ %/m (d-f), and $g = +1$ %/m (g-i). Black lines: a zero-crossing of the fields. Green lines: density peak of a nearby microbunch. In each plot, the line opacity corresponds to the value of the peak, normalized to its maximum value.*

The figure shows that, as expected, wakefields have in general a v_{ph} slower than v_b. Also, the effect of the density gradient is larger in the back than in the front of the bunch. The wakefields at the front have a similar behavior with all gradients. For $g = -1$ %/m, as the phase moves backwards the charge is expelled from the axis by the defocusing fields, and decreases significantly at $z \approx 5$ m (the green line disappears). The dephasing with respect to the bunch is the largest, because the effect of the SM growth, of the microbunch evolution along the plasma, and of the density gradient add to each other.

With $g = 0$ %/m the dephasing is smaller. The microbunches stay present for a longer distance, but eventually they are also defocused before the end of the plasma, at $z \approx 8$ m.

With $g = +1$ %/m, the phase in the middle of the bunch stays relatively constant, and the microbunch charge is largely preserved. At this ξ position, the density gradient in general mitigates the dephasing of the SM and microbunch evolution. Nevertheless, the effect of the gradient keeps increasing along the bunch, and the phase velocity eventually becomes superluminal, as seen in the back of the bunch. This is also unfavorable for the microbunches, and at $z \approx 9$ m, the charge starts decreasing.

Conclusion and summary

Introducing a density gradient in the plasma has a significant effect in the SM of a proton bunch and its further evolution in the plasma [6, 9, 10]. Simulations allow us to look into the details of the evolution of the system all along the plasma. A positive gradient reduces the detrimental effects of the slow-down of v_{ph} and leads to trains of microbunches that have more charge all along the plasma. However, v_{ph} becomes faster than the bunch velocity at large propagation distances, leading also to dephasing. It is a predicted that a density step [8] preserves the high amplitude of the wakefields for long distances, by preserving the charge that effectively drives wakefields, and is to be tested in AWAKE Run 2.

References

[1] A. Caldwell et al. (AWAKE Collaboration), NIM A, 829, 3-16 (2016)
[2] E. Adli et al. (AWAKE Collaboration), Phys. Rev. Lett. 122, 054802 (2019)
[3] M. Turner et al. (AWAKE Collaboration), Phys. Rev. Lett. 122, 054801 (2019)
[4] A. Pukhov et al., Phys. Rev. Lett. 107, 145003 (2011)
[5] C. B. Schroeder et al., Phys. Rev. Lett. 107, 145002 (2011)
[6] F. Braunmüller et al. (AWAKE Collaboration), Phys. Rev. Lett. 125, 264801 (2020)
[7] R. A. Fonseca et al., Lecture Notes in Computer Science 2331, 342-351 (2002)
[8] A. Caldwell and K. V. Lotov, Physics of Plasmas 18, 103101 (2011)
[9] A Gorn et al. (AWAKE Collaboration), 2020 Plasma Phys. Control. Fusion 62 125023 (2020)
[10] A. Petrenko et al., Nuclear Instruments and Methods in Physics Research A 829 (2016) 63–66