On control of the angular momentum of trapped electrons in tapered foam target

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Abstract: We have developed an analytical model to control the angular momentum of electrons trapped during relativistic laser-plasma interaction. To enhance the laser-plasma energy coupling we have used the foam target which regulates the near-critical density more efficiently in the medium. When the assumed incident laser pulse carrying large orbital angular momentum is incident on the target, the ponderomotive force and self-generated electromagnetic fields associated with the laser affect the electrons motion and transfer the momentum to the plasma electrons. The angular momentum associated with the energetic particle beams gives an additional degree of freedom which has potential applications in different fields such as condensed-matter spectroscopy and new electron microscopes.

Introduction

Laser-based plasma accelerators have stimulated great interest nowadays due to its compact and cost-effective benefits. With the use of ultra-intense laser technology, we can produce even high-energy electron beams in plasma [1]. The emittance of these high-energy electron beams has been studied in various laser-plasma experiments but only a few have unveiled the angular momentum associated with these beams [2]. The electron beams with high angular momentum have wide applications in condensed-matter spectroscopy and new electron microscopes [3,4]. Thaury et al. [5] were the first who successfully perform the experiment and established the evolution of the electrons’ angular momentum during the acceleration. However, due to experimental fluctuations the transferred angular momentum was uncontrollable in his experiment. Ju et al. [6] have used the circularly polarised laser pulse for the production of such electron beams.

In this work, we have controlled the angular momentum of particles by regulating the laser and plasma parameters by using Laguerre-Gaussian (LG) laser pulse. LG beam possesses central minima which can be produced by the conversion of Hermite-Gaussian (HG) modes [7] and have vast applications such as atomic trapping [8], atomic cooling etc. [9]. These beams
carry an orbital angular momentum as their poynting vector has an azimuthal component which results in helical wave front. These beams are capable of excitation of doughnut shaped plasma waves in nonlinear media which can focus the positrons efficiently.

Mathematical Model

A tapered foam target is irradiated by laser pulse which leads to ionization and hence, the formation of near-critical density plasma [10]. The heating of bulk electrons present in foam-attached target triggers the laser-acceleration and generate energetic electrons which shows better laser-plasma coupling as compared to usual plasmas. Then we launch first order linearly polarised LG laser pulse, with angular frequency ($\omega_0$) and wave number ($k_0$), in the generated near-critical density plasma. The electric field of linearly polarised LG laser pulse propagating in z-direction can be written in cylindrical coordinates ($r, \theta, z$) as:

$$E_L(r, \theta, z, t) = E_L(r_{b0}) \exp \left(-\frac{r^2}{b_0^2} + i \theta\right) L_0^1 e^{i(k_0 z - \omega_0 t) \hat{z}}$$

where $\theta$ is the azimuthal angle, $b_0$ denotes the laser spot size, $E_L$ is the laser amplitude, $r = \sqrt{x^2 + y^2}$ is the radial distance from the axis with transverse co-ordinates $x, y$ and $L_0^1 = 1$ is the associated Laguerre first-order polynomial with $p = 0$ radial index and azimuthal index, $l = 1$. Under the interaction of ultra-short laser pulse with relativistic plasma, the motion of electrons can be affected by spontaneous quasi-static self-generated electric field $\vec{E}_S$ and magnetic field $\vec{B}_S$. As the incident laser field is linearly polarised the axial component of self-generated magnetic field, evolved by the rotational current, comes to be zero. The dynamics of electrons can be calculated by using equation of motion as:

$$\frac{d\vec{p}}{dt} = -e\{\vec{E}_L + \vec{E}_S + \vec{v} \times (\vec{B}_L + \vec{B}_S)\}$$

Here $\vec{p} = \gamma m_e \vec{v}$ is the electron momentum having mass $m_e$ and velocity $\vec{v}$, $\vec{E}_L = E_{LR} \hat{r} + E_{L\theta} \hat{\theta}$ and $\vec{B}_L = \frac{-E_{Lr}}{v_{ph}} \hat{r} + \frac{E_{L\theta}}{v_{ph}} \hat{\theta}$, are the transverse electromagnetic fields of laser pulse with phase velocity, $v_{ph} = \frac{\omega_p}{k_0}$. Due to interaction of laser pulse with pre-formed plasma, ponderomotive force comes into existence that expels electrons and generates self-consistent accelerating field. Owing to the tapered geometry of the medium, the laser wave front confronts hollow cone which results in an open-mouth donut bubble that can trap electrons with transverse momentum

$$p_r(t) = \frac{-2m_e c a_L \omega_0^2}{(\omega_B - \omega_L)(3\omega_B + \omega_L)} \cos \theta \sin \omega_- t \cos \omega_+ t$$

(3)
where \( \omega_L \approx \left(1 - \frac{v_z}{v_p h}\right) \omega_0 \) is the laser frequency in electron moving frame, \( c \) is the speed of light, \( a_L = \frac{eE_L}{m_e c \omega_0} \) is the normalised laser amplitude, \( \omega_B \) refers to the betatron frequency of electron oscillation, \( \omega_- = (\omega_B - \omega_L)/2 \) and \( \omega_+ = (\omega_B + \omega_L)/2 \). The trapped electrons can acquire angular momentum due to azimuthal component of self-generated magnetic field and LG electromagnetic field. At resonance condition, \( \omega_B \approx \omega_L \), electrons will stay in the acceleration phase with helical line structure which suggests finite angular momentum of electrons in the z-direction expressed as

\[
L_z = \left(\frac{m_e c \omega_0^2}{4 \omega_0 \omega_L}\right) r \sin \omega_L t \sin \theta
\]

(4)

Here, \( \omega_p^2 = \frac{n_e e^2}{4m_e \epsilon_0} \) is the plasma frequency with electron density \( n_e \) and electric permittivity \( \epsilon_0 \).

**Results and discussion**

In Fig. 1, it has been shown that the transverse momentum for the typical resonant electron grows linearly with time and is oscillatory in nature.

![Figure 1: The time evolution of the transverse momentum \((p_r)\) for a resonant electron with \(a_L = 10\), \(\theta = 60^0\), \(b_0 = 10 \mu m\), \(z = 10 \lambda_p\), and \(\omega_L = 0.1 \omega_0\).](image1)

Figure 2: The angular momentum of resonant electron in the unit of \(\lambda m_e c\) at \(\omega_0 t = 200\) for \(b_0 = 1 mm\), \(\theta = 80^0\), \(\lambda = 10 nm\) and \(\omega_L = 0.1 \omega_0\).
Here, the incident laser field provides an external force while the self-generated fields are responsible for the restoring force to electron betatron oscillations. On the other hand, Fig. 2 suggests the variation of angular momentum of electrons with the laser frequency. As the frequency goes away from the resonance condition, the amplitude of angular momentum decreases. This is due to the fact that the maximum energy gain takes place at resonance condition. Hence, we can control the angular momentum of the trapped electrons by controlling the laser-plasma parameters as appeared in Eq. (4).

Acknowledgement
The authors, Sheetal Punia and Manish Dwivedi, acknowledges the Council of Scientific and Industrial Research (CSIR), Government of India, for providing financial support.

References: