Cylindrical vs toroidal Single Helical states in the low aspect- ratio RELAX device

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Introduction:
The RELAX device is a low aspect- ratio (with R/a=2 where R and a are the major and minor radius of the torus) Reverse Field Pinch (RFP) in which on-axis resonant helical modes with m=1 and n=4 (m and n being the poloidal and toroidal mode numbers respectively) are observed during the so called Single Helical (SH) states, i.e. states in which the plasma shows (at least in some time windows) an helical saturated dominant perturbation. These states have been predicted in the past by numerical 3D visco-resistive MHD simulations and afterwards detected in almost all RFP’s including RELAX.
The SH states can also be studied and characterized, as shown recently, within a cylindrical relaxation theory which assumes as global invariants the plasma volume integrated magnetic helicity plus a dominant mode helical flux weighted helicity [1].
In this paper we compare the predictions of the cylindrical relaxed states with the solutions that can be obtained by using the VMEC helical equilibrium solver and also with the states obtained by the 3D MHD MIPS code. Both the VMEC and MIPS solutions are fully taking into account the toroidal geometry, so it would be interesting to check to which extent the toroidal effects modify the cylindrically symmetric relaxed states.

Single helicity characteristics
In [2] a cylindrical model solving iteratively the problem of a RFP SH plasma equilibrium satisfying the Ohm’s law was studied, showing that the problem only admits an approximate solution, since the ohmic constraint is not exactly satisfied at the radius of reversal of the toroidal magnetic field.
This helical symmetric approximate solution was compared with the internal measurements of perturbed magnetic fields obtained in the RELAX experiment and a good agreement was found [1]. In particular the radial profiles of the radial, poloidal and toroidal perturbed
magnetic fields are well matched by the helical magnetic field as obtained following the procedure proposed in [2].

It was also shown in [1] that the cylindrical solutions should be shifted radially by the amount of the estimated toroidal Shafranov shift in RELAX, typically 20% of the plasma minor radius, i.e. 4-5 cm (a=25 cm), in order to match the measurements.

Here we extend the comparison to the helical equilibrium solution obtained with the code VMEC [3]. The result is shown in Fig.1 for the (1,4) harmonic of the magnetic field components, calculated following the procedure explained in [4]. We build two equilibria, one axisymmetric (‘ax’) and one helical (‘h’), with the same ideal, circular boundary, differing only for the safety factor profile definition. While the axisymmetric \( q_{\text{ax}}(\psi) \) profile (with \( \psi \) the poloidal flux) is monotonically decreasing, with \( q_{\text{ax}}(0) > 1/4 \), the helical profile has a maximum roughly at mid-radius, and \( q_{\text{h}}(\psi) < 1/4 \) for all \( \psi \). Of course, without any constraint for the equilibrium solution, there is some arbitrariness in the choice of the \( q_{\text{h}} \) profile. Now we express the helical equilibrium in terms of the axisymmetric one + perturbation, \( B_{\text{h}} = B_{\text{ax}} + \delta B_{\text{ax}} \) by means of a coordinate transformation between the related coordinate systems. As expected, this \( \delta B_{\text{ax}} \) perturbation has a dominant (1,4) Fourier component, which is plotted in Fig. 1 vs. the radius of the axisymmetric (toroidally shifted) surfaces.

The cylindrical and VMEC solutions are multiplied by a constant to approximately match the experimental values. Moreover, as mentioned, the cylindrical solution is plotted against a shifted radius to take into account the above mentioned Shafranov shift.

The VMEC solution matches the shape of the experimental data rather well, both near the axis and at the plasma edge. The profile of the main harmonic of \( \delta B_{\text{ax}} \) resembles the shape of a kink-like mode. On the other hand, the cylindrical (1,4) mode corresponds to a resonance at \( r/a \approx 0.3 \), and has tearing-like nature.

![Fig.1: Radial profiles of the radial, poloidal and toroidal magnetic field components from the experiment (with error bars), cylindrical and VMEC solutions.](image-url)
In Fig.2 we extend the comparison between the cylindrical code and the VMEC solutions. In Fig.2(a) the axi-symmetric safety factors, in (b) the helical and axi-symmetric VMEC q profiles as a function of the helical coordinate s and in (c) the helical flux functions (in red for VMEC and blue for the cylindrical solution) are shown. The values of F and Θ (see Ref.[1]) are -0.05 / 1.54 for the VMEC equilibrium and -0.015/ 1.64 for the cylindrical case. Once again the flux function (and radial magnetic field) of the cylindrical case are shifted in radius by 20% to mimic the toroidal shift. The q profiles differ mainly in the core region. The toroidal and cylindrical flux surfaces also differ especially because the VMEC solution produces a less indented and also radially narrower island-like structure.

3D nonlinear numerical simulations with the MIPS code have been also performed at shallow reversal. In Fig.3 the Poincarè plot of the 1/4 island structure and the radial profiles of the
dominant mode magnetic field components, also in comparison with the experimental measurements, are shown.

In MIPS code [5], the nonlinear resistive and compressive MHD equations are solved in a full-toroidal three-dimensional geometry by using the 4th-order finite difference and 4th order Runge-Kutta integration methods. All the variables are evaluated in a cylindrical \((R, z, \phi)\) coordinates with the numbers of grid points \((N_R, N_z, N_\phi)\)=(112,112,128). The initial condition of the simulation is given by the reconstructed equilibrium from the experimental data of RELAX by using the RELAXFit code[6]. The result reported in Fig.3 corresponds to the Case C of Ref. [7], which has \((F, \Theta)\)=(0.0, 1.7). The boundary condition is a fixed perfectly conducting wall at \(r=0.26\) m. Physical dissipation terms are included as constant resistivity \(\eta=1\times10^-5\) and viscosity \(\nu=8\times10^-4\).

Conclusions

Simulations with different numerical approaches of the helical states at low aspect ratio have been done and have been compared with the available experimental measurements in RELAX. The overall mode structure of the 1/4 dominant mode that can be deduced from a cylindrical code (corrected by the Shafranov shift), the VMEC helical equilibrium code all reasonably agree with the measurements. The MIPS simulations also qualitatively agree with the 1/4 island structure, as detected in RELAX, even if, it should be noted that in this numerical simulations the state with a dominant \(n=4\) structure is only transiently achieved and therefore can not be considered as an equilibrium for the system.