Aspect on equilibrium calculation of ITER configuration in Tokamaks

Tao Xu\textsuperscript{1*}, Richard Fitzpatrick\textsuperscript{1†}

\textsuperscript{1}Institute of Fusion and Plasma Research, key laboratory of advanced electromagnetic engineering and technology, Huazhong University of Science and Technology, Wuhan 430074, PRC.

\textsuperscript{2}Institute for Fusion Studies, University of Texas at Austin, Austin, TX 78712, USA.

Abstract

In this work, we have constructed an ITER configuration, which is generated by currents flowing within the plasma and currents flowing in external coils. The plasma current density takes the form $j_{p}(r, z) = -ar - bR^2/r$ or other H-mode current profile inside the plasma, and is zero in the surrounding vacuum. We use Green’s function method to compute the plasma current contribution, together with a homogeneous solution to the Grad-Shafranov equation, to construct the full solution. Matching with the constant boundary condition on the last closed flux surface is performed to determine the homogeneous solution. Then the total solution in the full space is obtained. We can also obtain the value of external coils current by the homogeneous solution.

Equilibrium computation is crucial for the design and operation of magnetic fusion devices. The equilibrium magnetic configuration in magnetic

\* E-mail address: xutao@hust.edu.cn,

\† E-mail address: rfitzp@farside.ph.utexas.edu
The confinement devices is determined by the Grad-Shafranov (GS) equation. The GS equation is a nonlinear elliptic partial differential equation, which is usually solved by numerical computation. In 1968, Solov’ev [1] proposed simple linear stream functions, and got analytic solution for the Grad-Shafranov equation. Solov’ev’s equilibrium configurations are useful for the benchmarking magnetohydrodynamics equilibrium codes [2] as well as stability analysis of toroidal axisymmetric tokamaks.

We use cylindrical coordinates $r, \varphi, z$ to describe toroidally axisymmetric plasma configurations. It is well known that the poloidal flux function $\psi(r, z)$ satisfies the Grad-Shafranov equation

$$
\frac{\partial^2 \psi}{\partial z^2} + r \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial \psi}{\partial r} = -r j_{\varphi}, \quad j_{\varphi} = rp' + \frac{I_A I'_A}{r},
$$

(1)

where $j_{\varphi}$ is the longitudinal current density along the cross section of the plasma, $p$ is the plasma pressure, and $I_A$ is the net poloidal current function. Both $p$ and $I_A$ are free functions of $\psi$. The prime denotes differentiation with respect to $\psi$. We can easily extend our analysis to take such equilibria into account by writing

$$
\psi = a_1(r^2 - R^2) + b_1 z + c_1(r^2 - R^2)z + \frac{AR^2}{2} (1 + C \frac{r^2 - R^2}{R^2})z^2 + \frac{a + b - A}{8} (r^2 - R^2)^2 - \frac{b - (1 - C)A}{24R^2} (r^2 - R^2)^3.
$$

(2)

Let us consider the case that $a_1 = 0.01$, $b_1 = 0.01$, $c_1 = -0.01$, $a = 0.2$, $b = -4a/5$, $C = 9/5$, $A = 0.034a$, $R = 6$. The poloidal magnetic surface function in plasma region is

$$
\psi = 0.01(r^2-36)+0.01z-0.01(r^2-36)z+0.00017(1+\frac{r^2-36}{36})z^2+\frac{415}{100000}(r^2-36)^2.
$$

(3)

The analytic solution (3) of the magnetic surface function is shown in figure 1.

The full solution of Grad-Shafranov equation has two parts which can be written as $\psi = \psi_G + \psi_h$. Here $\psi_G$ is the vacuum solution by plasma current density, $\psi_h$ is the homogeneous solution. The magnetic surface function $\psi_G$ calculated by Green’s function method is

$$
\psi_G(r, z) = \int_{JR} G(r, z; r', z')j_{\varphi}(r', z')dr'dz',
$$

(4)
The poloidal magnetic flux generated by currents flowing in distant magnetic field coils takes the form [3]

\[ \psi_h(r, z) = \sum_{N=1}^{N \neq 1} c_N \psi_N(r, z), \]

where the \( c_N \) are arbitrary coefficients. When \( N \) is even,

\[ \psi_N(r, z) = \sum_{n=0,N/2-1} A_N^n r^{N-2n} z^{2n}, \quad \frac{A_N^n}{A_N^{n-1}} = -\frac{(N/2 + 1 - n)(N/2 - n)}{n(n - 1/2)} \]

for \( 1 \leq n \leq N/2 - 1 \), with \( A_N^0 = 1 \). On the other hand, when \( N \) is odd,

\[ \psi_N(r, z) = \sum_{n=0,(N-3)/2} A_N^n r^{N-2n-1} z^{2n+1}, \quad \frac{A_N^n}{A_N^{n-1}} = -\frac{(N/2 + 1/2 - n)(N/2 - 1/2 - n)}{n(n + 1/2)} \]

with \( A_N^0 = 1 \). The coefficients, \( c_N \), that characterize the homogenous solution, (5), are determined by demanding that \( \psi_p(r, z) + \psi_h(r, z) = (1 - \eta) \psi_X \) on the control surface \( \psi = (1 - \eta) \psi_X \). The simulated result of the magnetic surface function by our method is shown in figure 2.

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**References**


Figure 1. $a=0.01, b=-4a/5, c=9/5, A=0.034a, R=6$. The magnetic configuration is calculated by method of Solovev.

Figure 2. $a=0.01, b=-4a/5, c=9/5, A=0.034a, R=6$. The magnetic configuration is calculated by our method.