Possibility of anomalous emission at half-integer pump wave frequency harmonics in the X2 ECRH experiments

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A while ago it was revealed theoretically [1,2] that in presence of a non-monotonic plasma density profile, originating due to the magnetic island or the density pump-out effect at on-axis electron cyclotron resonance heating (ECRH), the low-power-threshold absolute two-upper-hybrid (UH)-plasmon parametric decay instability (TPDI) of a pump microwave can occur in the hundred-kilowatt X2 ECRH experiments. In the general case its excitation leads to the generation of both UH wave localized in the vicinity of the local maximum of a plasma density profile and of the non-trapped UH wave. Depending on the dominating saturation mechanism (the cascade of secondary decays or the pump depletion), according to the theory predictions [1,2], from 10% up to more than 60% of the pump power can be transferred to the daughter UH waves as a result of TPDI. This instability manifests itself in anomalous backscattering effect [3] leading to emission of radiation at frequency down-shifted by several GHz in respect to the pump wave. According to the theory [1] correctly reproducing the frequency spectrum and intensity of the backscattering signal, this emission is generated due to nonlinear coupling of parametrically excited UH waves.

In the present paper we consider theoretically a possibility of anomalous emission in X2 ECRH experiments of electromagnetic waves possessing a larger frequency shift, namely, of the pump frequency half-integer harmonics. The experimental conditions when such an emission is possible are determined and the corresponding power is estimated.

The pump power nonlinearly deposited during the primary decay into the UH daughter waves is divided in two practically equal parts between the UH wave trapped in the vicinity of the local maximum of the non-monotonic density profile and the UH wave non-trapped there. In this situation one could expect the strong emission of electromagnetic waves at half the pump wave frequency caused by the second UH wave. However such an emission will be possible only if the non-trapped UH wave converts into the X-mode propagating out of the UH resonance (UHR) in the high-magnetic field-side direction, as it is shown in Fig.1 for the primary decay occurring in the magnetic island of TEXTOR tokamak for the plasma density in the local minimum of the profile very close to the UH resonance value for the half pump frequency...
In this case the primary TPDI power threshold is equal to $P_{0}^{th} = 127.8$ kW and the anomalous absorption efficiency is estimated well above the threshold at the level of 12% [4]. The X-mode crosses the ECR surface and leaves the plasma at the high-field side. Neglecting the ECR absorption of the X-mode which is known to be small at the perpendicular propagation in modest temperature plasma of middle-scale toroidal devices and taking into account the power balance in the decay we can estimate the power of the radiation in the first EC harmonic range associated with this X-mode. For the experimental conditions of Fig. 1 it is

$$P_{\alpha/2}^X = \alpha P_0 \approx 0.05 \cdot P_0^2.$$  

The larger part of this power is reflected from the device wall in the form of the X-mode and finally is absorbed after conversion in the UHR. However a smaller part of the power, characterized by cross-polarization factor $\beta_{XO}$, is reflected in the form of O-mode, which is partly absorbed in the ECR and then leaves the plasma. Taking into account the O-mode optical depth $\Gamma_{O1}$ we obtain the estimation

$$Q_O \approx 0.5 \alpha \beta_{XO} \exp \left(-\Gamma_{O1}\right) P_0.$$  

Taking into account that for the conditions of the TEXTOR off-axis ECRH experiment $\Gamma_{O1} \approx 2$, and assuming $\beta_{XO} = 0.01$ we get for the case under consideration $Q_O \approx 8 \cdot 10^{-5} P_0$. Thus at the maximal microwave power of 600 kW one can expect to see at the low field side of tokamak the 50 Watts O-mode emission at half the pump frequency. This emission is observable only in the narrow density range when the UH density for half the pump frequency is slightly lower than plasma density in the profile local minimum, however the decay with the upper branch of the non-localized UH wave (electron Bernstein wave) is still possible. Below this density the TPDI leads to excitation of only trapped waves or not possible at all if $\omega_0 > 2\omega_{UH}(x_{\text{max}})$. Above this density range the intensive emission of sub-harmonic $\omega_0/2$ is not possible because the parametrically excited UH wave propagates towards the UHR converts into the electron Bernstein wave and finally is absorbed in plasma. At higher plasma densities the sub-harmonics emission is possible only in higher frequency ranges. Namely, the nonlinear coupling of the trapped daughter UH wave possessing much higher

![Fig. 1. Dispersion curves of primary daughter waves generated in the course of TPDI; 1 - non-trapped UH wave, 2 - trapped UH wave, the dimensionless UH frequency $f_{UH}^2/(f_0/2)^2$ in the magnetic island is given by the thick solid curve.](image)
amplitude and the pump wave can lead to the plasma emission in the third EC harmonic range or at the \(3\omega_0/2\) harmonic of the pump frequency. This effect is similar to the phenomenon predicted and observed in the laser fusion experiments [5]. This possibility is illustrated in Fig. 2 where the dispersion curve of primary trapped UH wave at frequency \(\omega_2\) (solid line) along with \(k_{ax} + \omega_a / c\), \(\omega_a = \omega_0 + \omega_2 \approx 3/2\omega_0\) (dashed line) is shown. In points, where these lines intersect, the Bragg scattering conditions are fulfilled and generation of the \(3\omega_0/2\) harmonic radiation takes place. It happens when the following conditions hold

\[
\omega_c \left( x_{\text{max}} \right) \sqrt{\frac{n_e \left( x_{\text{max}} \right)}{\delta n_e}} < \frac{5\omega_0}{2} < \frac{c}{l_I} \sqrt{\frac{\delta n_e}{n_e \left( x_{\text{max}} \right)}},
\]

where \(\delta n_e \equiv n_e \left( x_{\text{max}} \right) - n_e \left( x_{\text{min}} \right)\), \(n_e \left( x_{\text{max}} \right)\) and \(n_e \left( x_{\text{min}} \right)\) stand for the density value in the local maximum and minimum of the profile accordingly (Fig. 2). In the case of decay shown in Fig. 1 only the primary UH wave appears to be able generating the wave at frequency \(3\omega_0/2\) propagating outwards. The microwave signal at this frequency received by an antenna on the low-magnetic field-side of the toroidal device can be calculated with the help of reciprocity theorem in the following form

\[
A(\omega_a) = \frac{1}{4} \int j_{\text{nl}}(\omega_a, r) E^+ (\omega_a, r) \, dr,
\]

The integration here is carried out over the whole plasma volume and the normalized to unite power receiving antenna beam \(E^+ (\omega_a, r)\) with the waist \(w_a\) in a vicinity of the O-point of the magnetic island (assumed in the near-zone of the receiving antenna) is defined as \(E^+ = \mathbf{e}_y \sqrt{8 \frac{1}{c} w_a^2} \exp \left( i \frac{\omega_a}{c} x - \frac{y^2 + z^2}{2 w_a^2} \right)\). The nonlinear current describing the excitation of the X-mode at frequency \(3\omega_0/2\) being generated due to the interaction between the primary UH wave and the pump wave is given by the expression

\[
\left( \mathbf{e}_y \cdot j_{\text{nl}} (\omega_a, r) \right) = i \frac{\omega_c}{4\pi} \frac{E^*_0}{\omega_0 H} c q_s^2 a_s (y, z) \psi_2 (x),
\]

where

Figure 5. The dispersion curves of primary trapped UH wave (solid line) and \(k_{ax} + \omega_a / c\) (dashed line) The density profile in the magnetic island is given by the thick solid curve.
The backscattering signal power $p_s$ equal to $|A(\omega_a)|^2$ is given by the expression

$$p_s = \frac{P_0}{H^2 w^2 w_0^2} \int q_{2x}^2 a_2(y,z) \varphi_2(x) \exp \left[ i \left( \frac{\omega_a + \omega_b}{c} x - \frac{y^2 + z^2}{2} \left( \frac{1}{w^2} + \frac{1}{w_0^2} \right) \right) \right] \frac{dr}{4\pi}$$

(2)

Taking into account that in the saturation regime of TPDI the daughter UH wave localization region is much wider than $w_0 \approx 2w$ and using its level estimated in [4] we obtain an estimation of the $3\omega_b/2$ emission power received by the antenna from direct integration of the expression under the integral (2). At the TEXTOR magnetic island control experiment plasma parameters considered in the paper we come to the estimate $T_{3/2} = \frac{P_s}{P_0} \approx 5 \cdot 10^{-5}$. For the maximal power utilized in this experiment it results in 30 W of received $3\omega_b/2$ pump harmonic emission.

Thus the nonlinear coupling of the daughter UH waves with the pump could lead to the measurable level of the plasma emission at the $3/2$ harmonic of the pump, in the way similar to that occurring in the laser driven inertial fusion experiments [5].

It should be mentioned that the predicted signal is a result of linear Bragg scattering of the pump wave off the parametrically driven UH wave, whereas the anomalous emission in the vicinity of the gyrotron frequency observed in [3], according to [1], is produced by a nonlinear effect of UH waves coupling. In this sense the $3\omega_b/2$ pump harmonic emission could be considered as a more direct source of information on the level and spectrum of UH waves excited in plasma as a result of low-threshold two-plasmon parametric decay instability.

Calculation of the plasma emission at the third EC harmonic was supported by RSF grant 16-12-10043, whereas estimation of the emission at the first EC harmonic was funded by the Ioffe Institute.