A real case of complex network controllability: the NIO1 ion beam source

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1. Introduction

A key component of ITER is the Neutral Beam Injector (NBI) heating system, which is based on two injectors designed to deliver 33MW of power to the plasma. The achievement of the NBI nominal parameters (40 A of negative ions H\textsuperscript{-}/D\textsuperscript{-} to be accelerated to 1 MeV and then neutralized with 60\% efficiency) and its reliable operation are challenging tasks, which are the objective of an extensive international research and development programme. Among the issues raised by the project, its controllability is particularly interesting. Reliable operation of NBIs is the result of several processes, mutually interacting in an often non-linear way. NBIs can then be looked at as an example of a complex system, whose controllability can be investigated and tackled by novel tools offered by the theory of complex systems and their controllability [1]. The theory has been applied to the network that models generation, extraction and acceleration of negative ions in the negative ion beam source NIO1 [2], operating at Consorzio RFX. Processes and relative links have been combined in a model whose robustness has been verified. For this latter purpose NIO1 constitutes an ideal case, as it is currently operating without caesium and the neutralizer, so that a limited number of nodes and links are sufficient to describe the system. A subset of processes has been identified as the driver processes whose key role is thus highlighted and can be used to guide future in-depth analysis of the involved physics.

2. Description of the model

Complex network theory is based on the assumption that systems can be described by a Linear Time-Invariant (LTI) approximation $\dot{x} = Ax + Bu$, where the vector $x(t)$ represents the state of the system variables and $u(t)$ the state of external inputs, $A$ and $B$ are the constant matrices containing the coefficients that model the system evolution. As recently pointed out [1], the system can also be seen from a graph-theoretical point of view: each process or phenomenon inside the system is associated to a state variable and becomes a node of the
system. If node \( i \) has influence on node \( j \) within the system, then there is a link pointing from \( i \) to \( j \). In graph theory, matrix \( A \) is called *adjacency matrix*, and represents the system wiring diagram: that is, entry \( a_{ji} \) is nonzero if a link is pointing from node \( i \) to node \( j \). Matrix \( B \) is called *input matrix* and has nonzero \( b_{ik} \) if input \( k \) is acting on node \( i \). By definition a system is controllable when it can be driven from any initial state to any final state in finite time [3]. It has been proven [4] that steering a subset of the system nodes, namely the MSDN (Minimum Set of Driver Nodes), is enough to drive the whole system towards any state in its phase space. Each network has several MSDN of the same cardinality. In other words, several subsets of driving processes are available but they always consist of the same number of nodes. The maximum matching of the network is the maximum subset of links of the network that do not share start or end nodes. It has been demonstrated [1] that by finding one of the many maximum matchings of a network, a corresponding MSDN can be obtained.

With reference to the NIO1 experiment, a total of 40 processes (the nodes of the network) were identified and a graph-theoretical model was generated [5] [6]. Processes occur on one of the following regions of NIO1: the ion source, where the plasma is generated and which is bounded by the plasma grid (PG), the extraction and the acceleration regions, associated to the corresponding extraction (EG) and acceleration (AG) grids, and the beam line region, where the beam transport occurs. Fig. 1a shows a graphical rendering of the complex network of processes and links, while in fig.1b the physical location where they occur in the beam source is shown. Links between processes were therefore identified and the corresponding adjacency matrix \( A \) and input matrix \( B \) were built. Multiple maximum matchings of NIO1 network were then found and the number of driver nodes was established to be four [5] [6] [7]. Thus, the entire system can be driven by driving only four of the forty total processes. One of the many possible maximum matchings is shown in fig. 1c. Several maximum matchings of the NIO1 graph were enumerated with the key assumption that if a node recurs in several different MSDN this may suggest some intrinsic importance of the associated physical process [7]. The most common by far is the driver node set involving the following processes: plasma drifts in ion source, density of \( H_0 \) between plasma grid and extraction grid, deflection of \( H^- \) ions between plasma grid and extraction grid, and density of \( H_2 \) inside the vessel. By adjusting these values, the whole system can in principle be steered where desired. However, the first three processes are especially difficult to control from the outside, and it would be practically more feasible to focus on different nodes.
The most favourable driver node set involves mostly nodes that are easy to control: source pressure, density of atomic $H_0$ in the gap between plasma grid and extraction grid, density of $H_2$ in the vessel plus the always present deflection of $H^-$ ions between PG and EG. The last process has been found to play a key role in meniscus formation and in the deflection of the beam [7]: its presence among the driver nodes suggests further investigations on the topic.

3. Validation of the model

The LTI system $\dot{x} = Ax + Bu$ (whose matrices $A$ and $B$ represent the system network) can still be solved and the simulation can be compared to experimental data from NIO1 in order to validate the model. In principle such a model is limited in two ways: firstly, its linearity reduces its applicability to the neighbourhood of a steady state; secondly, the matrix does not attempt to map physical quantities and only measures relations between nodes on a normalized $[0, 1]$ scale. Thus, instead of numerical correspondence between model results and experimental data, coherence on trends and parameter interdependence should be expected. Still, the overall evolution of the system in the neighbourhood of a steady state can be observed in order to spot similarities or deficiencies in the model. An input signal was injected into the system and its effects on the relevant nodes were measured. The NIO1 data collected through several experimental campaigns spanning a wide range of parameters [9] have therefore been used to further validate the model by comparing them with the solutions of the LTI system. Six major inputs were selected for the comparison: source pressure $p_{\text{source}}$ (with cryopump turned on and off), Radio Frequency (RF) power injected in the plasma source $P_{\text{RF}}$, plasma grid filter current $I_{\text{PGF}}$, extraction voltage $V_{\text{EXT}}$ and acceleration voltage $V_{\text{ACC}}$. For each parameter the following outputs were considered: extracted current $I_{\text{EXT}}$, divergence $\theta$, beam aiming error $\delta$, beam particle energy $\epsilon$ and the ratio between extracted electron and ion currents $j_e/j_{H^-}$. For each scan, a simulation was run with NIO1 graph model to mimic the experimental data and look for correlations.
Figure 2 NIO1 experimental data (dots, left y-axis) compared with NIO1 LTI system solutions (lines, right y-axis). Since the model is built in arbitrary units a direct comparison with the experimental data is not straightforward. To overcome this, both experimental and model data are normalized to their maximum value (see below); units of measurement of the bottom x-axis are indicated, units of measurement of top x-axis are arbitrary. (left) Scan in source pressure, $V_{\text{ACC}} = 2.5\, \text{kV}$, $V_{\text{EXT}} = 350\, \text{V}$, $P_{\text{RF}} = 1.2\, \text{kW}$, $I_{\text{PGF}} = 10\, \text{A}$. (right) Scan in $I_{\text{PGF}}$ (plasma grid filter current), $V_{\text{ACC}} = 4\, \text{kV}$, $V_{\text{EXT}} = 450\, \text{V}$, $P_{\text{RF}} = 1.4\, \text{kW}$, $P_{\text{source}} = 0.75\, \text{Pa}$ (beam energy data not available).

The overall result shows a good correspondence between model and data in pressure scans (both with and without cryopump), filter current scans, extraction voltage scans and RF power scans (see Fig. 2), where trends of beam energy, $I_{\text{beam}}$ and $j_e/j_H$– ratio are correctly predicted. Fig. 2 shows how predictions for the extracted current and beam energy are quite accurate. Nevertheless, because of its linearity, the model fails in precisely following the $j_e$–$/j_H$– curve. As the model can describe the overall physical response, these results corroborate what has been said above on controllability, and prompt to further investigate, both by measurements and modelling, nodes that were reported as driver by the analysis. This is especially true for the meniscus zone. It is worth noticing that the agreement between predicted and effective behaviour of NIO1 also validates the initial choice of processes and links which to some extent could be considered arbitrary.

4. References