Different dynamic regimes of stimulated electron-cyclotron emission from mirror-confined non-equilibrium plasma

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Introduction

Studies of cyclotron instabilities have led to the plasma cyclotron maser paradigm, which explains a rich class of phenomena of coherent plasma emission [1]. In particular, electron cyclotron instabilities caused by resonant interaction between energetic electrons and electromagnetic waves are typical for open magnetic traps with electron cyclotron resonance (ECR) plasma heating [2]. One of the present applications is related to a development of ECR ion sources. Particle ejections, which are inherent to the burst regime of the cyclotron instability, cause oscillations of the plasma potential and the beam current accompanied with a significant decrease of the average ion charge [3]. Recently we demonstrated experimentally that tuning of the ECR position in a MHD-stable minimum-B open magnetic trap allows switching from the generation of periodic bursts of electromagnetic radiation to a continuous-wave (cw) low-power regime of emission [4]. In this way we eventually avoid non-desirable effects of bursts and improve the ion source performance. Similar systems have been previously studied in the context of space cyclotron masers in planet magnetospheres and other astrophysical objects [5]. However, a laboratory experiment is characterized by a very different source of fast electrons, thus the existing theories need to be reconsidered [6].

Maser equations

Assuming that the cyclotron instability evolves slowly compared to the bounce- oscillations of resonant electrons in a trap and a frequency spectrum of wave turbulence is narrow compared to the electron-cyclotron frequency, a self-consistent evolution of particles and waves may be described by the bounce-averaged quasilinear equations [6]:

$$\frac{\partial F}{\partial t} = \frac{\partial}{\partial \kappa}\left(\left(ED + D_0\right)\frac{\partial F}{\partial \kappa}\right) + J, \quad \frac{\partial E}{\partial t} = \left(\int_0^\infty \int_{\kappa_c}^1 K \frac{\partial F}{\partial \kappa} d\kappa d\upsilon - \nu\right) E\,, \tag{1}$$

where $F(t,\kappa,\upsilon)$ is the electron distribution function over electron velocity υ and pitch-angle $\kappa = \upsilon_\perp \sqrt{B_{\min}} / \upsilon\sqrt{B}$ in the trap center, $\kappa_c \propto \sqrt{B_{\min}/B_{\max}}$ is loss-cone boundary, J is a source of non-equilibrium electrons. The quasi-linear diffusion coefficient ED takes into account the electron scattering into the loss-cone by unstable waves with average energy E; D_0 account for other mechanism of electron losses such as Coulomb collisions, for simplicity we assume $D_0 = E_0 D$. The wave energy $E(t)$ is determined from the balance equation, in which the instability grow rate is proportional to $\partial F / \partial \kappa$, and ν stands for wave dissipation rate.

One can seek a solution of (1) as series over eigenmodes $\phi_n(\kappa)$ of the quasilinear operator. To be definite, we consider the strongest interaction case with the extraordinary wave at the fundamental ECR harmonic, then $D/\kappa \approx const$ and $K/\upsilon^4\kappa^2 \approx const$. Introducing

$$\int_0^\infty F\upsilon^4 d\upsilon = \sum_n f_n(t)\phi_n(\kappa), \quad \int_0^\infty J\upsilon^4 d\upsilon = \sum_n j_n(t)\phi_n(\kappa), \quad \frac{\partial}{\partial\kappa}\left(D\frac{\partial\phi_n}{\partial\kappa}\right) = -\mu_n\phi_n, \tag{2}$$

one can rewrite equations (1) as an infinite set of ordinary differential equations [4]:

$$\frac{df_n}{dt} = j_n - \mu_n(E+E_0)f_n, \quad \frac{dE}{dt} = \left(\sum_n k_n f_n - \nu\right)E, \tag{3}$$

To determine source J of non-equilibrium particles let us consider resonant electrons heated by an external monochromatic radiation under ECR condition. Due to interaction with the heating waves, such electrons redistribute along the curves of quasilinear diffusion, $2\gamma - \upsilon_\perp^2\gamma/c^2 = \text{const}$, where γ is the relativistic factor and $\upsilon_\perp$ is calculated at the position of "cold" ECR. Assuming that the electrons accelerated from very low energy, they all belong to the same curve $2\gamma - \upsilon_\perp^2\gamma/c^2 \approx 2$: we obtain some universal relation $\kappa^2 = 2B_{\min}/(1+\gamma)B_{ECR}$ between the electron pitch angle and energy. At some κ^* this curve crosses another line corresponding to quasilinear diffusion induced by excited unstable waves. Apparently such crossing may be treated as δ-like distribution over pitch-angles, i.e. $j_n = J_0\phi_n(\kappa^*)$. Thus, laboratory plasma masers are characterized by a source with a broad spectrum over ϕ_n allowing simultaneous excitation and interaction of many eigenmodes in (3).

Dynamic regimes: theory and experiment

In spite of high dimension, equations (3) have only two steady-state solutions: one corresponding zero wave energy, i.e. a regime with no maser generation, and one corresponding to non-zero wave energy, i.e. a regime of cw generation. Switching between these two modes can be demonstrated even if only one mode $n = 1$ is taken into account in (2). One can easily find a bifurcation criterion for such system: if $j_1 k_1 > \nu\mu_1 E_0$ then the zero-wave-solution becomes unstable while the second solution corresponds to a stable regime of stationary generation.

The situation becomes less trivial when we take into account other modes. It is possible that both stationary solutions are linearly unstable, then, from topological considerations, our system must have a stable limit cycle corresponding to a regime of burst generation. The simplest case of such behavior may be studied within the adiabatic approximation, $df_n / dt = 0$ for all $n \geq 2$, which is justified by a fast growth of μ_n with the mode number ($\mu_2 \approx 10\mu_1$, $\mu_n \propto n^2$). We again may consider the only differential equation for f_1 assuming $f_n \approx j_n E / \mu_n$ for $n \geq 2$. Then, the condition for the existence of a stable limit cycle can be found as $k_1 j_1 > 0$ and $\sum_{n\geq 2} k_n j_n / \mu_n < 0$. Physically this condition implies that the burst regime is realized when the lowest (non-adiabatic) mode is destabilizing for waves while all higher (adiabatic) modes act as a non-linear absorber [4].

Next, we may consider first n^* modes as non-adiabatic, assuming $f_n \approx j_n E / \mu_n$ for $n > n^*$. This generalization is needed to describe the case of "weak quasilinear diffusion" which corresponds to limited strength of source J typical for a laboratory experiment. The boundary n^* between adiabatic and non-adiabatic modes may be determined self-consistently using the following condition: $j_1 k_1 / \nu \approx \mu_1^2 / \mu_{n*}^2 \propto (n^*)^{-4}$ [6]. For $n^* > 1$ one more bifurcation is possible, namely, a pair birth of stable and unstable limit cycles while the stability of the stationary point of the system does not change. Physically this may be understood as follows. During the developed burst regime, there is a deep modulation of the wave power $E(t)$, the system spends much of the time in a state where E is close to its minimum value and the quasi-linear diffusion is weakened. In this case, the effective boundary between the adiabatic and nonadiabatic modes is defined from $j_1 k_1 / \nu \approx \mu_1 / \mu_{n*} \propto (n^*)^{-2}$, i.e. it shifts towards larger mode numbers and, as a consequence, Lyapunov's exponent for (3) can change sign. The range of parameters where it happens corresponds to the unstable limit cycle. The described set of bifurcations is illustrated in figure 1 which shows the results of numerical solution of system (3) for different values of the control parameter κ^*. One can see a hysteresis typical for "hard birth" of two limit cycles. The stable limit cycle corresponding to the burst regime merges with the unstable one and disappears at $\kappa^* = 0.88$, as a result, the system abruptly switches from burst to cw regime *for increasing* κ^*. The region of attraction of a stationary point collapses to zero and the stationary point becomes unstable at $\kappa^* = 0.835$, as a result, the system abruptly switches from cw to burst regime at this point *for decreasing* κ^*. More details may be found in [6].

Recently the predicted hysteresis dynamics was found experimentally at the JYFL Ion Source (University of Jyvaskyla). In experiment we control the dynamic regimes by varying the source strength J determined by the ECRH power supporting the discharge and parameter

$\kappa^* \propto \sqrt{B_{\min} / B_{\mathrm{ECR}}}$ determined by the external magnetic field. Figure 2 shows the results of observations of dynamic modes of generation of stimulated ECR radiation for different values of ECRH power for the increasing and decreasing magnetic field. Observed coincidence of the theoretical predictions with the experiment shows the adequacy of the cyclotron maser model, and allows one to search for more efficient modes of operation of the ECR ion source. The work is supported by RFBR (project no. 19-02-00767).

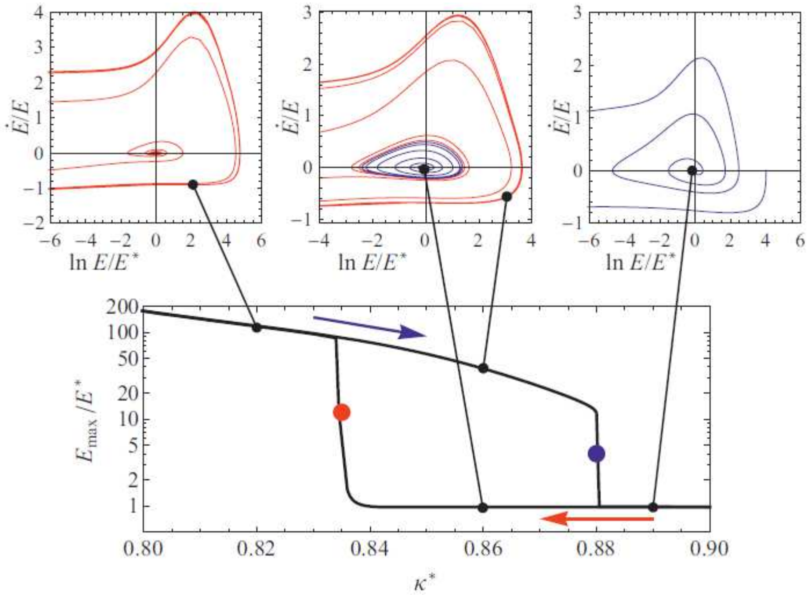

Fig. 1. Characteristic phase trajectories and the peak wave energy in a settled oscillatory regime as a function of the ECR position for fixed particle source strength and dissipation. Numerical solution of (3) for the first sixteen modes being taken into account. Andronov-Hopf bifurcations are indicated by the large points.

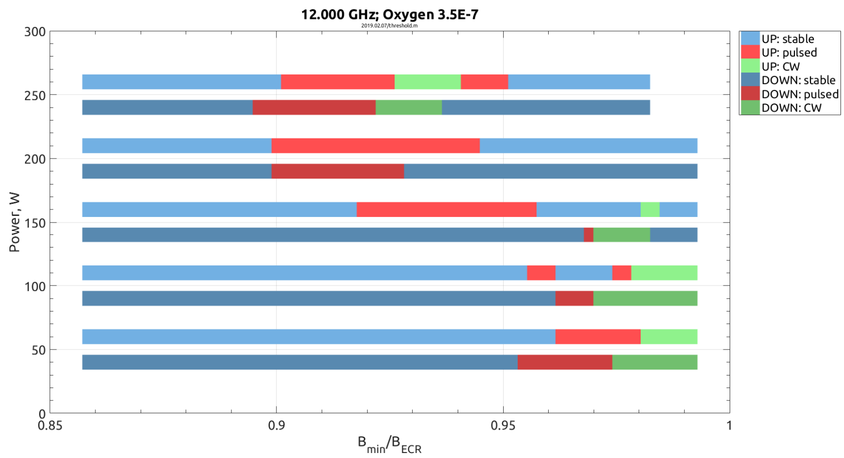

Fig. 2. Different regimes (stable, cw, pulsed) of plasma emission measured at JYFL Ion Source for different experimental parameters for increasing and decreasing confining magnetic field.

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