Channeling of Neutral Beam Injection Power into Radiofrequency Waves

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The concept of alpha power channeling has been proposed by N. Fisch and collaborators as an efficient tool to improve the performance of fusion reactors, delivering the fusion alpha power into radio frequency waves, which are then absorbed by ion species [1]. This scheme of ion heating by alpha particles is much more efficient, in terms of fusion performance, than the standard scheme of electron heating by collisional absorption. Alpha channeling into ion Bernstein waves (IBW) by Doppler-shifted inverse nonlinear Landau damping at half-integer cyclotron resonance has been theoretically predicted recently [2], suggesting that Deuterium neutral beam (NB) injection power can be also channeled into IBW. NB channeling into IBW has been indeed recently observed [3]. We discuss here a possible effect of channeling into IBW waves of Deuterium NB power injected in D(20%)-T(76%) tokamak plasma with ⁹Be minority (1%). The IBW are coupled by mode conversion of fast magnetosonic waves (FW), launched from the low field side. The NB-IBW channeling effect is provided by inverse Landau damping at Doppler-shifted fundamental ion cyclotron resonance \( \omega = \Omega_D - k_\parallel v_\parallel \), thanks to the free energy available from an inversion of population of the fast deuterons in the perpendicular velocity space. It might occur transiently, during the evolution of the D⁺ distribution function towards the slowing down equilibrium. For sufficiently large loss rate of the fast deuterons the inversion of population can persist in steady state conditions. The IBW, amplified by the channeling effect, are damped by ⁹Be⁴⁺ ions near their fundamental cyclotron resonance, which is located on the high field side of the mode conversion layer, in the region of IBW propagation. The mode conversion scenario is shown in Fig.1 (left), based on the set of tokamak plasma parameters of Table I. We assume that 10 MW of ICRH power are coupled at the operating frequency \( f = 23 \) MHz with parallel refractive index \( N_{\parallel 0} = k_{\parallel 0} c / \omega = -3 \) and 1.0 MeV Deuterium beams with 25 MW total power are injected at about 48° with respect to the magnetic field in the IBW region. For 80% MC efficiency, as the FW damping is negligible, 8 MW IBW power is available (Tab. II).

<table>
<thead>
<tr>
<th>( R/a (m) )</th>
<th>( I_p (MA) )</th>
<th>( B_0 (T) )</th>
<th>( n_e (m^{-3}) )</th>
<th>( T_e (keV) )</th>
<th>( T_i (keV) )</th>
<th>( q_{95} )</th>
<th>( \delta )</th>
<th>( \kappa )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.0/0.85</td>
<td>2.6</td>
<td>3.3</td>
<td>10²⁰</td>
<td>5.0</td>
<td>7.0</td>
<td>4.0</td>
<td>0.3</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Tab. I. Plasma parameters of the NB-IBW channeling scenario. Parabolic profile for the density and squared parabolic profile for the temperatures are assumed. The plasma composition is \( X_D = 0.2, X_T = 0.76, X_{Be} = 0.01. \)
We model the time evolution of the distribution function \( f_D(v,t) \) concerning the deuterons provided by the NB source, near the magnetic axis, based on the kinetic equation

\[
\frac{\partial f_D}{\partial t} = \frac{1}{v_{\perp}} \frac{\partial}{\partial v_{\perp}} v_{\perp} D_{rf} \frac{\partial f_D}{\partial v_{\perp}} + s_D + \mathcal{K}(f_D) - v_l f_D \tag{1}
\]

\( D_{rf} \) is the quasilinear diffusion coefficient in velocity space, \( s_D \) is the source of the \( D^+ \) ions produced by the NB injection, \( \mathcal{K} \) indicates the collision operator and \( v_l \) is the rate of \( D^+ \) losses, which is assumed constant.

The collision operator in velocity coordinates \((v, \mu)\), where \( \mu = v_\parallel / v \) is the cosine of the pitch angle, is approximated \([4]\) as

\[
\mathcal{K}(f_D) \approx \frac{1}{\tau_s v^3} \left\{ v \frac{\partial}{\partial v} \left[ (v^2 + v_\parallel^2) f_D \right] + \frac{v^3}{2} \frac{\partial}{\partial \mu} \left[ (1 - \mu^2) \right] \frac{\partial f_D}{\partial \mu} \right\} \tag{2}
\]
Here \( v_c \) is the critical velocity for equal rates of energy exchange in the collisions with the background electrons and ions and \( \tau_s \) is the slowing down time. The other parameters are \( Z_2 = \sum_i Z_i^2 X_i / Z_1 \) and \( Z_1 = \sum_i Z_i^2 X_i n_D / m_i \), where \( X_i = n_i / n_e \) and the sum on \( i \) extends over all ion species with electric charge charge \( +Z_i e \) and mass \( m_i \).

We average the quasilinear diffusion coefficient on a magnetic surface, and obtain

\[
D_{rf} \approx \frac{e^2 E_0^2}{m_B^2} \frac{R}{h_p \omega} \int_1^2 \left( k_1 v_\perp / \omega \right) e^{-\left( v_\parallel - v_{10} \right)^2 / 2 \sigma_0^2} \tag{3}
\]

Here \( E_0 \) is the IBW electric field amplitude, \( v_\parallel = (\omega - \Omega_D(R)) / k_{\parallel 0} \), \( J_1 \) indicates the Bessel function of the first kind of order 1, \( h_p \) is the poloidal half-length of the section of the magnetic surface area illuminated by the IBW, which we assume symmetric with respect to the equatorial plane and with almost constant major radius coordinate \( R \).

The Gaussian shape in parallel velocity space is obtained assuming Gaussian shape of the IBW power spectrum vs. the parallel wavenumber. The source is assumed Gaussian in energy and anisotropic in velocity space, with linear pitch-angle dependence, namely

\[
s_D = S_0 \exp\left[ - (E - E_0)^2 / \Delta E^2 \right] (1 + \mu) v^{-1} F_s^{-1} \tag{4}
\]

where \( S_0 \) is the number of ions produced from the beam per unit of time and volume and \( F_s = (\Delta E / E_0)^{3/2} v \left[ 1 + erf \left( E_0 / \Delta E \right) \right] \) is a normalization factor. The relevant boundary conditions are \( \partial f_D / \partial v_\perp = 0 \) at \( v_\perp = 0 \), as required by the cylindrical symmetry, and \( \lim_{v_\parallel \to \infty} f_D = 0 \). The initial condition is \( f_D = 0 \) at \( t = 0 \). To obtain numerical solution of the kinetic equation we use a fractional step algorithm [5]. We split the rf-induced diffusion and the collisional operator and alternate the solutions of the relevant differential equations, namely \( \partial f_D / \partial t = D(f_D) + s_\alpha / 2 - (v_\parallel / 2) f_D \) and \( \partial f_D / \partial t = K(f_D) + s_\alpha / 2 - (v_\parallel / 2) f_D \).

The former equation is solved by Crank-Nicolson method on a rectangular mesh in the cylindrical velocity space, the second is solved analytically [2]. For plasma and rf parameters of Tab. I, and assuming \( R / h_p = 12 \), \( \sigma_\parallel = 0.05 v_a \) and \( \Delta E = 0.1 \) MeV, an inversion of population in the perpendicular velocity space at the steady state is obtained considering a loss rate \( v_l = 32 \) Hz (Fig. 2). In this scenario, the slowing down frequency is \( v_s = \tau_s^{-1} \approx 3.7 \) Hz, so that \( v_l \) is larger than the threshold \( 3v_s \) found in [2] for steady state inversion of population and power channeling. In Fig. 1 (right) we show the power exchanged between IBW and the beam ions as a function of the time, which is calculated by the expression

\[
P_{IBW \to D^+ / NB} = -4\pi \int_{-\infty}^{+\infty} dz \int_0^{\infty} dx \: x^2 D(x, z) \frac{\partial u}{\partial x} \tag{5}
\]
Here we define $x = v_1/v_o$, $z = v_\parallel/v_o$, $D(x,z) = D_{rf}(x v_o, z v_o) v_o^{-2} v_s^{-1}$, $u = f_D n_D^{-1} v_o^3$, $v_o = \sqrt{2} E_o/m_D$, $n_D = S_o/v_s$. Negative values of the ratio $P_{IBW\to D^+}/P_{NB}$ indicate that the corresponding fraction of NB power has been delivered to the IBW.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Time evolution of the normalized distribution function of the fast D$^+$ ions produced by the NB injection in the presence of the IBW for plasma parameters of Table I, shown by the Contour plots of $\log_{10}(f_D n_D^{-1} v_o^3)$ at different times. The abscissa is the normalized parallel velocity $v_\parallel/v_o$, the ordinate is $v_\perp/v_o$.}
\end{figure}

For the scenario considered here about 14\% NB power is channeled into IBW power in steady state conditions. We have applied the model also to the ITER-scaled parameters. Preliminary numerical results confirm that about 10-15\% NB power can be delivered from 1 MeV D$^+$ ions produced by the neutral beam injection to IBW.

References


