Modelling of non-linear Edge Harmonic Oscillations and the effect of
non-axisymmetric magnetic coils

G. Bustos¹, J. Graves¹, A. Kleiner¹, D. Brunetti², W.A. Cooper¹

¹ École Polytechnique Fédérale de Lausanne (EPFL), Swiss Plasma Center (SPC), Switzerland
² Istituto di Fisica del Plasma IFP-CNR, Via R. Cozzi 53, 20125 Milano, Italy

The Quiescent High confinement operational mode (QH-mode) offers an excellent alternative to the well known H-mode due to the absence of ELMs while keeping similar plasma performance. A feature of QH-mode is that a continuous long wavelength perturbation called Edge Harmonic Oscillation (EHO) saturates non-linearly, enhancing particle transport in the edge thus avoiding impurity accumulation in the core plasma. Given the importance of EHO in QH-mode, it could be useful for control and reproduction of robust QH-mode scenarios to assess how non-axisymmetric resonant magnetic perturbations couple with EHO. This could help for example to modify the main characteristics of QH-mode discharges while keeping constant the key elements needed to achieve QH-mode. With that in mind, the effect of global toroidal mode seeding in the vacuum magnetic field is investigated.

In the frame of ideal MHD, in equilibrium and in the absence of flows, the non-linear saturated states can be obtained with the VMEC code [1], which minimises the total energy of the system with the constraint of fulfilling the equilibrium equation \( \mathbf{J} \times \mathbf{B} = \nabla P \). Here, \( \mathbf{J} \) is the current density, \( \mathbf{B} \) the magnetic field and \( P \) the plasma pressure. In the minimisation process VMEC takes into account the vacuum magnetic field, which is calculated externally through the Biot-Savart law from a set of coils represented by infinitesimally thin wires. In this work, we have used a set of JET-like coils, including an up-down symmetric version of the Error Field Correction Coils (EFCCs) to apply the resonant magnetic perturbations. The effect of the small up-down asymmetry was investigated and concluded to be weak for low applied current to the EFCCs ( <200kA). The EFCCs consist of four external coils evenly distributed in the toroidal direction, capable of producing a magnetic perturbation with \( N=1,2 \) (\( N \) is the toroidal mode number of the externally applied perturbation). We specify 96kA on each of the coils, which will produce a perturbation on the background magnetic field of \( \delta B_\phi \sim 2 \times 10^{-2}T \) at the plasma edge, where \( \delta B_\phi \) is the amplitude of the toroidal component of the seeded mode \( N \).

We model nonlinear saturated states in a QH-mode-like equilibrium, with a pressure pedestal and a spike in the edge current density profile 1a. Such spike models qualitatively the bootstrap contribution in the low-collisionality regime and produces an extended region of low shear localised at the pedestal. As done in [2], we focus our attention to \( n=1,2 \) modes (\( n \) is the plasma
toroidal mode number). To calculate the nonlinear plasma displacement it is useful to define a 2D axisymmetric surface, obtained from simulating the same equilibrium conditions as in the 3D case but allowing only \( n=0 \) modes in the VMEC fourier expansion. We define the nonlinear radial plasma displacement \( \xi_r \) as the distance from the 3D to the 2D flux surface in the direction normal to the 2D surface [3] (see figure 1b). As the magnetic field is frozen to the plasma, we can also calculate the perturbed magnetic field \( \delta \mathbf{B} \) by subtracting the field in every point of the 3D surface minus the field from which the fluid was displaced in the 2D surface (in the appropriate set of coordinates). Finally, in order to make comparisons with analytical models, the plasma displacement and magnetic field perturbation are mapped to straight field line coordinates and fourier analysed [3].

We then calculate the nonlinear states of the equilibrium described above (figure 1a), varying the value of the safety factor at the edge \( q_a \), the safety factor at the plateau \( q_* \) and the value of beta at the pedestal top \( \hat{\beta} \). The main results are summarised in figure 2. As in [2], we separate the 3D perturbed states into "pressure driven" and "current driven" modes. For the current driven modes (figure 1a), we observe 3D perturbations resembling those observed in QH-mode discharges (EHO) with most unstable mode \((m,n)=(4,1)\) in a region \( 3 < q_a < 4 \), in agreement with the external kink theory. Applying a global \( N=1 \) perturbation results in a amplification of the mode (6.3% increase at the peak position) and a slight broadening on the...
Figure 2: Normalised nonlinear amplitude of the most unstable mode as a function of (a) $q_a$ (current driven mode with $(m,n)=(4,1)$) or (b) $q_*$ (Pressure driven mode with $(m,n)=(5,1)$). (c) shows the radial profile of the cylindrical toroidal component of $\delta B$ for pressure driven cases with different applied perturbations and values of $q_*$. Blue curves represent the plasma displacement when no resonant magnetic perturbation is added to the system. Green/red curves show the plasma displacement when applying a N=1/2 resonant magnetic perturbation by adding current to the EFCCs.

parameter space ($q_a$) in which EHO are observed. Applying a global N=2 perturbation has a very weak effect on the amplitude of the perturbation and slightly reduces the $q_a$ parameter space for triggering EHO. For the pressure driven modes (figure 2b), we observe EHO with most unstable mode $(m,n)=(5,1)$ in the region where $q_*$ is close to $m/n=4$, in agreement with the infernal mode theory. Applying a global N=1 perturbation has a larger impact on the amplitude of the plasma displacement ($\sim 24\%$ at $q_* = 4$) and the parameter space where EHO are observed is increased significantly. This indicates a strong coupling of the infernal mode driver with the externally applied perturbation. Applying a global N=2 perturbation has a again week effect on the amplitude of the plasma displacement, and reduces slightly the parameter $q_*$ for triggering EHO. Figure 2c shows the cylindrical toroidal component of the perturbed magnetic field ($\delta B \cdot \hat{\phi}$) for pressure driven EHO with different external perturbations and values of $q_*$. In accordance with figure 2b, the plasma response to a N=1 mode is to enhance the perturbed field (green solid line in figure 2c), while for a N=2 mode is to reduce the perturbed field (red solid line in figure 2c). Nevertheless, when applying a N=2 perturbation, the most unstable mode becomes $(m,n)=(0,2)$ (red dashed lines in figures 2b and 2c). This N=2 enhancement in the perturbed field has however little effect on the amplitude of the plasma displacement, as seen in figure 2b.

Finally, it can be noted in figure 2b that the amplitude of the plasma displacement when no external perturbation is applied and with $q_* = 3.98$ is roughly the same as when a global N=1 perturbation is applied and $q_* = 3.90$. We can compare the toroidal component of $\delta B$ of these
two equilibrium. As can be seen in figure 2c, $(\delta \mathbf{B} \cdot \hat{\phi})$ is almost identical for the two cases (solid blue and dashed green lines). Note also that $\delta \mathbf{B} \cdot \hat{\phi}$ is of the same order as the magnetic perturbation induced by the EFCCs in vacuum. Therefore, we can formulate the following hypothesis: if the necessary magnetic perturbation to excite the mode is not provided by the mode drivers (weak shear and pressure gradient), then this perturbation can be added externally, and will couple with the plasma mode in order to amplify its excitation. As the mode driver gets smaller, the coupling with external magnetic perturbations gets weaker, and so the amplitude of the plasma displacement is reduced. This hypothesis would also explain why in the current driven case there is only a small broadening of the parameter space ($q_a$) in which EHO are observed. For external kinks, the mode driver vanishes for $q_a > m/n$, so the additional external perturbation cannot couple with the instability driver. On the contrary, in the infernal case the mode driver does not vanish as $q_\ast$ gets farther from the resonance surface, it just gets weaker. More simulations and a rigorous analysis (rather than mere speculation) are needed to corroborate this hypothesis.

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References

