Magnetic nulls from the topological perspective

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Magnetic nulls in 3 dimensional fields appear in the solar corona, in planetary fields and in fusion concepts such as the polywell and Field Reversed Configuration (FRC). These points where the field vanishes are topologically stable and are hotspots for magnetic reconnection. We analyze these nulls, their motion and coalescence in topologically non-trivial analytical vector fields.

Being the zeroes of a continuous vector field, magnetic nulls are governed a topological index theorem. This theorem states that the index of an isolated null equals the degree of the mapping from a surface enclosing the null to the unit sphere. If a surface encloses more than one null, then the degree of the mapping from this surface to the sphere equals the sum of the indices of the nulls enclosed.

We relate the properties of these mappings to the eigenvectors of the matrix of partial derivatives, which is the conventional method of analyzing 3D null points [1], and show that type A nulls carry negative topological index and type B nulls positive.

We describe the construction of the isotropic field, whose integral curves are lines where the field points in one unique direction. As a configuration changes, the nulls must move along these curves of constant direction. As such, this foliation of space provides an elegant method of tracking the nulls. We demonstrate this by tracking the locations of the null points around a localized, finite energy analytical vector field as we change the magnitude and direction of an applied guide field in a configuration that is topologically similar to a planetary field embedded in a guide field, or the field of an FRC configuration.

References