A new analytic solution to the collision free plasma equation with warm ions

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Determining plasma flow to boundaries in cases when ion mean energies are comparable to, higher than or even considerably above the temperature of the main/bulk electrons (usually Maxwellian) is an important but rather demanding task which still lacks satisfactory theoretical description and characterization. In this context, a particular problem of interest here concerns modeling of the ion velocity distribution function (VDF) \( f_i(v, q) = 2 \int_0^{q_W} \frac{1}{E(q')} \sqrt{2(q' - q + v^2/2)} \) as the formal solution of \( v \frac{\partial f_i}{\partial x} + E \frac{\partial f_i}{\partial v} = s(v^2, q) \), i.e., the Boltzmann equation written for a collision-free symmetric plane discharge (of length \( 2L \)), with the Maxwellian ion source \( S_i(x, v) \) (see [1, 2] and references therein), where the following physical and normalized quantities have been used: length \( \bar{x} \equiv x \), ion and electron VDFs \( \leftrightarrow v_{i,e}, u_{i,e} \), densities \( n_{i,e} \equiv n_{i,e} \), velocities and directional mean velocities \( \frac{v_{i,e}}{c_{se}}, \frac{u_{i,e}}{c_{se}} \leftrightarrow v_{i,e}, u_{i,e} \), temperatures \( T_{i,e} \leftrightarrow T_{i,e} \), potential \( \frac{e\Phi}{kT_{e0}} \leftrightarrow \Phi = q \), electric field \( E = -d\Phi/dx \), and source \( LS(v^2, q)/n_0 \leftrightarrow S(v^2, q) \), with \( n_0 = n_i(0) = n_e(0) \), \( T_{i,e,0} = T_{e,i}(0) \) the values at the plane of symmetry of the discharge, \( e \) the positive elementary charge, \( k \) the Boltzmann constant, \( c_{se} \equiv (kT_{e0}/m_i)^{1/2} \) the "artificial" ion-sound speed, and \( m_i \) the ion mass. The key quantity for calculating the ion VDF is the electric field, which in the limit \( T_n \gg T_{e0} \) takes the rather inconvenient, i.e., hardly usable, form

\[
\frac{1}{E(q)} = \frac{2T_n + 1}{\sqrt{2\pi T_n}} e^{(\beta - 1)q - (1 + \frac{1}{2\gamma_E})q^2} \left[ \frac{1 + \frac{1}{2\gamma_E}}{\pi e^{-(1 + \frac{1}{2\gamma_E})q^2}} \int_0^{q_s} \frac{\sqrt{t(t - q_s)}}{q - t} e^{(1 + \frac{1}{2\gamma_E})t} dt - \frac{I_0\left(-\left(1 + \frac{1}{2\gamma_E}\right)q_s\right)}{\ln\left(\frac{16T_n}{\gamma_Eq_s}\right)} \right],
\]

where \( q_s \) stands for the quasi-neutral plasma-edge boundaries of integration (which are different from the wall potential \( q_W \) calculated numerically and tabulated in [2] for a large number of \( 0 < T_n < 10^2 \)), \( I_0(z) \) is the familiar zero-order Bessel function, and \( \gamma_E = \exp(C_E) = 1.78107 \) (with the Euler-Mascheroni constant \( C_E = 0.57721 \) ...). Strictly speaking, the above quasi-analytic expression for the electric fields has been derived under the condition \( T_n > -\Phi \) but fits the experimental ones perfectly only for \( T_n > 3 \) [3]. In contrast, we have modified it into a highly reliable new analytic expression holding for any \( T_n > 0.3 \) which, moreover, is very simple and readily employable. Its validity is confirmed via comparison with the "exact" numerical parametric solutions obtained by varying the potential profile (parameter \( \beta \)) and the source temperature \( T_n \).

References