Anti-symmetric plasma fluid models with exact discrete conservation

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We construct fluid plasma models that combine physical consistency and numerical stability with a simple and flexible implementation [1]. This is achieved by exploiting the anti-symmetric property of the plasma flow operator, \( \int dV \phi ( \nabla \cdot v + v \cdot \nabla ) \psi = - \int dV \psi ( \nabla \cdot v + v \cdot \nabla ) \phi \), which results in exact conservation. Since this expression has an equivalent discrete analog, computer implementations of fluid models using our representation automatically inherit the conservation properties of the continuous system. Additionally, it can be shown that the plasma velocity generates infinitesimal rotations of the fluid element, implying local time reversibility, e.g. existence of a discrete inverse. The anti-symmetric equations are written using generalized moments, \( \sqrt{n}, \sqrt{n}v, \sqrt{p} \), related to conserved quantities \( n, K = (1/2)mv^2, U = (3/2)p \), that ensure mass and energy positivity. Altogether, these properties lead to an intrinsically stable code implementation, provided that a few restrictions on the choice of numerical scheme are satisfied. We demonstrate this by applying our methodology to the Braginskii two-fluid model, the magnetohydrodynamics (MHD) equations, and also more computationally efficient drift-ordered models. The conservation properties are verified using representative cases such as single seeded blob dynamics and the Orszag-Tang vortex, obtaining high-fidelity simulation results with negligible dissipation. – This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Fusion Energy Sciences, Theory Program, under Award no. DE-FG02-95ER54309.

References


Figure 1: (left) Snapshot of plasma density and (right) traces of mass, momentum, and energy error in 2D MHD simulation of the Orszag-Tang vortex