Double tearing modes in the presence of anti-symmetric shear flow

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The linear properties of both even and odd double tearing modes (DTMs) in the presence of plasma shear flow are studied based on a reduced resistive MHD model in slab geometry. It is found that for the symmetric shear flow, the linear growth rate of even eigenmode of DTM is almost independent of the strength of shear flow, while the odd eigenmode decreases with the shear flow strength. However, for anti-symmetric shear flow, the linear growth rates of even (odd) eigenmode of DTM decrease (increase) with increasing the strength of shear flow. Moreover, in the small wavenumber regime, the growth rate of the even eigenmode is larger than that of the odd eigenmode, while the growth rates of two kinds of eigen states coalesce with each other (the same growth rate and opposite frequencies) when the wavenumber exceeds a critical value Kc. It is demonstrated that Kc decreases with decreasing resistivity for a fixed separation between two resonant surfaces Xs, while decreasing Xs raises the critical value of Kc for a fixed shear velocity.

I. INTRODUCTION

Resistive tearing mode, a kind of magnetohydrodynamic (MHD) instability, can arise in solar flares, substorm in the earth magnetosphere, and current disruptions in laboratory devices like tokamaks. Coupling and interaction between tearing modes on adjacent resonant surfaces with the same helicity play an important role in disruption process, referred to as double tearing mode (DTM). This instability is stronger than a single tearing mode (STM), whereby modes with high mode numbers or short wavelength may become dominant.

Plasma flows, approaching the Alfvén velocity, can greatly modify the stability criteria of STM. Tearing instability can be either enhanced or suppressed by shear flow plasma parallel to the reconnection magnetic field depending on the plasma viscosity, the magnetic shear, and the strength of flow shear. Recently, it is recognized that shear flows can influence the linear and nonlinear developments of DTM in a certain degree through two kinds of stabilizing mechanisms: (1) flow shear on an individual rational surface has an indirect effect on the DTMs by locally stabilizing/destabilizing the single tearing reconnection on the
rational surface; (2) flow shear between the both rational surfaces has a directly stabilizing effect on the DTMs by reducing the mutual driving strength of the DTM islands.

In this work, we discuss in detail the linear and nonlinear properties of the DTM instability numerically in the present of symmetric and anti-symmetric shear flow within the framework of a reduced MHD model:

\[ \partial_t \nabla^2 \phi + [\phi, \nabla^2 \phi] = [\psi, \nabla^2 \psi], \]

\[ \partial_t \psi + [\phi, \psi] = \eta \nabla^2 \psi, \]

where \( \psi \) and \( \phi \) are the magnetic flux function and the stream function, respectively. The double current sheets and the shear flow profiles used in this work are shown in Fig. 1.

![Equilibrium magnetic field, symmetric and anti-symmetric shear profiles.](image)

II. LINEAR SPECTRA OF THE DTM

The growth rate of both even and odd DTMs for each shear velocity is indicated with solid and dashed curves respectively in Fig 2(a). Note that in the absence of shear flow, the growth rate of even DTM (shown by solid black curve) scales as \( ky^{(2/3)} \) in the non-constant-\( \psi \) regime, however the growth rate of odd DTM (shown by black dash curve) is very small and has a weak effect on the main results. It is seen that growth rate of even DTMs decreases/increases with increasing shear velocity in the small/large \( ky \) regime. On the other hand, increasing shear flow raises the growth rate of odd DTMs, but this growth rate is still less unstable and can be ignorable. Fig. 2(b) reveals the real frequency of both even and odd eigenstates. The frequency of the even DTMs and odd DTMs is equal and their corresponding curves lie on each other, because of the fact that the symmetric shear profile makes the absolute value of flow velocity on the resonant surfaces become equal.
The eigen structures of the even and odd DTMs for the case without flow (as marked in Fig. 2(a) with A and B) indicated that there is a strong coupling between both resonant surfaces for even DTMs. This coupling makes the even DTMs grow stronger than the odd DTMs, which has a weak coupling between both resonant surfaces.

For external anti-symmetric shear flow, the behavior of the instability is found to be more complex than that of the symmetric shear flow discussed above. Indeed, as shown in Fig. 3(a), by increasing the shear velocity, both even and odd DTMs are excited. The linear growth rate decreases with increasing shear velocity for even DTMs, while it increases with increasing shear velocity for odd DTMs. For the fixed anti-symmetric shear flow profile, the features of the linear DTMs in the low ky regime are different from those in the high ky regime. In the low ky regime, growth rate of even DTMs (solid curves in Fig. 3) are much larger than those of odd DTMs (dashed curves in Fig. 3), while the real frequencies of both even and odd DTMs are almost equal to zeros, as indicated in Fig. 3(b). However, in the high ky regime, the growth rate of even DTMs are the same as those of odd DTMs (solid and dashed curves are overlapped in this regime), while the real frequencies of even and odd DTMs are opposite due to the anti-symmetric shear flow profile. Moreover, since the coupling effect of the even DTM between two resonant surfaces becomes weak when ky increases, the critical parameter $K_c$, separating the low and high ky regimes, decreases with increasing the shear flow strength. The results obtained by the eigenvalue solver indicate that
the growth rate of odd DTM grows in $k_Y < K_c$ regime, coalesced with that of even DTM in the presence of anti-symmetric shear flow.

The linear eigenmode structures of even and odd DTMs in the presence of anti-symmetric shear flow for $k_Y = 0.05 < K_c$ and $k_Y = 0.7 > K_c$ (corresponding to points C, D, and E in Fig. 3(a)) are analyzed. In the case of $k_Y = 0.05$ (corresponding to points C, D marked in Fig. 3(a)), the real and imaginary parts of eigenmode for even DTMs have a stronger coupling than the eigenmode of odd DTMs, causing the growth rate of even DTMs to grow stronger than odd DTMs. For the $k_Y = 0.7$ (corresponding to point E marked in Fig. 4(a)), the eigenstructures of two eigenmodes with the same growth rate and opposite frequencies are mirror symmetry. They are respectively the distorted even and odd DTMs with anti-symmetric shear flow.

III. CONCLUSION

In this work, the linear stability of DTM in the presence of symmetric and anti-symmetric shear flow has been studied.

(1) For the symmetric shear flow, it is found the growth rate of even eigen state is more unstable than odd eigen state, while both eigen states have the same frequency. It is also found that, with or without symmetric shear flow, the growth rate scales as $k_Y^{2/3}$, which is the same as the analytical scaling in the DTM configuration.

(2) For the anti-symmetric shear flow, however, the effect of shear velocity becomes more prominent. For the low shear velocity, the corresponding growth rates of the DTM eigen states individually grow, and then join together in the non-constant-psi regime. The point that the two eigen states meet each other, named critical wave vector $K_c$, is shifted by changing the shear velocity and distance between the resonance surfaces. Increasing flow strength to 0.5, the peak of growth rate decreases nearly 40%. More importantly, the critical wave vector $K_c$ will eventually cross the peak point of the growth rate and move into the constant-psi regime. The $k_Y^{2/3}$ scaling gradually is decreased with no clear scaling in this phase.

(3) It is found that for a fixed shear velocity, $K_c$ decreases with decreasing resistivity. However, decreasing $X_s$ raises the value of $K_c$ for a fixed shear velocity. For a fix value of flow strength and $X_s$, the linear spectrum of DTM is broadened as resistivity increases. Similarly, it is found that by decreasing $X_s$ for a fix value of flow strength and resistivity, the linear spectrum of DTM is broadened in the large $k_Y$ regime.