Self-injection of multiple electron microbunches into a beam-driven plasma bubble

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Generation of multiple micro-bunches of electrons during wake-field acceleration were discussed for the case where electrons are self-injected in consecutive buckets behind the driving beam [1] or in a single bubble [2, 3]. Short (sub 10 fs) electron bunches can also be produced with the help of velocity bunching cavities as well [4], but their spatial separation is defined by the repetition rate of the laser used to trigger the electron ejection from the cathode. In this work we show that electron self-injection into plasma bubbles in the space-charge dominated regime has a periodic feature, providing comb-like electron bunches. We will show with help of 3D PiC VSim simulations that the train of the electron micro-bunches with ∼10 fs temporal period is self-injected into a bubble driven by a relativistic electron beam under certain conditions. The number of electron bunches, their durations, period and temporal structures are basically defined by the gradient of the plasma density down-ramp and by the total charge of the driver.

In uniform under-dense plasma the accelerating structure, driven either by laser pulse or e-beam, has a phase velocity equal to the velocity of the driving beam. In the case of laser pulses the propagation velocity depends on the plasma density. The laser diffraction can cause expansion of the bubble, which leads to the reduction of the phase velocity and as a result injection of electron bunch [5, 6]. The situation is different in the case of electron driving beams, where the phase velocity is practically defined by the energy of the driving beam. In the case of 10 GeV beams there are no electrons with velocity larger than the driving beam velocity. One way to control the velocity of the tail of the bubble is to use plasma with variable density, for instance decreasing density profile of the background plasma, i.e. a density down-ramp [7, 8]. Recently it was found that continuous injection of electrons is possible when a few millimeter long down-ramp is applied resulting in a long electron bunch with small emittance [9]. In the present work we consider much longer density ramps and higher charge of the driving beam, which leads to a different regime of electron self-injection [10].

In the blow-out regime electrons located closer to the propagation axis of the driving beam acquire higher radial momentum and they can reach the same radial position in the back of the
bubble (BOB). Basically they fall back due to the attracting force of the ion cavity and they cross the axis of symmetry or get injected depending on the longitudinal acceleration experienced in the shell of the bubble. If we neglect the azimuthal magnetic field generated by the BOB the problem can be considered one dimensional and the condition of injection is:

$$e(\Delta \phi) > m_e(\gamma_b - \gamma_0)c^2,$$  

(1)

where $\Delta \phi$ is the potential difference across the BOB, $\gamma_b$ is the Lorentz factor of this region of the bubble and $\gamma_0$ is the initial relativistic factor of electrons entering the BOB, which depends on the charge of the driving beam ($Q_b$). If the electrons gain enough energy from the longitudinal field their velocity can be higher than the phase velocity of the bubble (which is the velocity of the BOB, $v_b$) and get injected inside the bubble. If the longitudinal profile of the accelerating field is known, then Eq. (1) can be written in the following form:

$$\gamma_b - \gamma_0 < -\frac{e}{m_ec^2} \int E_x dx = \frac{e}{m_ec^2} \int \frac{v_e}{v_b - v_e} E_x d\xi,$$  

(2)

where $E_x$ is the longitudinal electric field, $\xi = x - v_b t$ is the coordinate of electrons in the co-moving frame and $v_e = dx/dt$ is the electron velocity. If the plasma density is not constant, decreases in space the plasma wavelength ($\lambda_p = (c/e) \sqrt{m_e e_0/n_e}$) and the length of the bubble increases and $v_b < c$ also changes slowly in time. In our theoretical study we consider the following longitudinal profile of the density down-ramp:

$$n_e = n_0 \left[ \rho + (1 - \rho) \frac{1 + \cos(\pi x/L_n)}{2} \right],$$  

(3)

where $\rho = n_1/n_0$, $n_1$ is the density after the ramp and $x$ goes from 0 to $L_n$, which we call here density scale length. Over the ramp the density decreases by a factor of four ($\rho = 0.25$), thus ideally at the end the bubble size is two times larger than at the beginning of the ramp. Using Eq. (3) one can calculate $v_b$ [10] and $\gamma_b = 1/\sqrt{1 - v_b^2/c^2}$ which is shown in Fig. 1, where for sake of simplicity the velocity of driving beam is equal to the speed of light. It can be seen that around the middle of the density down-ramp the injection is possible because of the low values of $\gamma_b$. If the electrons gain high enough energy $\gamma > \gamma_b$ then they get injected and it happens earlier in the case of shorter scale lengths. At the end of the ramp the injection stops because the plasma becomes uniform again.

In one dimension the injection is continuous but in a two-dimensional cylindrical symmetric system ($z$ and $r$ coordinates) the electron accumulation in the BOB can generate strong azimuthal magnetic field ($B_\theta$). Due to relativistic effects the electrons spend a long time in the
Figure 1: Lorentz factor of the back of the bubble as a function of propagation distance for different plasma profiles.

BOB where the current density continuously increases up to the point when self-magnetic field converts the longitudinal momentum into radial one. The azimuthal magnetic field suddenly drops and a significant amount of electrons leave the BOB radially, they get repelled by the space charge field of accumulated electrons.

Figure 2: Electron spatial distribution (a) and momentum-space distribution (b) near the crossing region of the bubble. The black color indicates the injected electrons ($v_x > v_b$) and the color code corresponds to the longitudinal momentum of radially repelled, ejected (or crossing) and injected electrons.

The process of transversal ejection is illustrated in Fig. 2. The electrons shown with blue color have small radial momentum and can not reach the middle of BOB thus they get repelled and contribute to the shell of the second bubble. The green electrons have higher momentum and they can get accelerated longitudinally and eventually get injected or ejected. The electrons with large momentum are shown with red color and their destiny (ejection or injection) is decided by the actual value of magnetic field or, with other words, by the peak density of the BOB. It is clearly seen from the momentum-space trajectories that the red electrons acquire high radial momentum before ejection. The condition of transversal ejection is reached when the electron gyro-radius is equal to the radius of the BOB: $r_g = R_b$, which is equivalent to $\gamma m_e c/(eB_\theta) \approx \lambda_p$. 

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where $B_\theta$ is defined by the peak density in the BOB ($n_p$). In Ref. [10] we derived a simple expression which expresses the condition for ejection: $n_p > 2m_e\gamma_0$, which is used to qualitatively explain the bunched injection observed in simulations.

![Figure 3: Cross section of electron density in bubbles generated by different driving beam charge and plasma density scale lengths.](image)

In Fig. 3 results obtained from 3D particle-in-cell simulations are shown where the driving beam charge and the length of the down-ramp are changed, the parameters which influence most the injection mechanism. From pictures 3a and 3b it can be seen that higher driving beam charge results in earlier injection because of higher $\gamma_0$ of electrons. The start of self-injection is delayed also by the longer density scale length, as it is seen in Fig. 3c, which is also in agreement with the model based on Fig. 1. The exact scaling for the periodicity and size of the micro-bunches requires more investigation. If the density scale length is short all electrons reaching the BOB can be injected thus the current density does not reach the critical value, thus the injection is continuous [9]. The fact that the energies of micro-bunches are not the same emerges the possibility of developing polychromatic X-ray sources or even gamma-ray sources via Thomson scattering, as it is discussed in [3].

References