Influence of stiff temperature profile on island stabilization by RF heating

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Summary The nature of turbulent transport in tokamak plasmas results in temperature profiles that are called resilient or stiff, and the stabilization of magnetic islands by a localized heat source is expected to be extremely sensitive to the stiffness strength. Theoretical expectations are verified with nonlinear simulations, showing a good agreement and confirming the enhanced stabilization efficiency due to large profile stiffness when the power used for the control is small compared with the heating power producing the equilibrium profiles. Heat sources that are present in the island region before the RF heating is applied contribute to reduce the island size, but at the same time, they severely damp the control capability.

Heat and particle transport in tokamaks are generally dominated by turbulent processes that are triggered above some critical gradient, following theoretical [1], and experimental works [2]. This issue is of interest not only for the understanding of the energy confinement time, but also in the context of magnetic island studies, since the flattening of temperature profile inside an island is expected to reduce locally the turbulent transport [3]. This damping of turbulent processes at the O-point of magnetic island has also been deduced from experimental observations in stellarators as well as in tokamaks [4].

Analytical model The turbulent heat diffusivity is modeled as

\[
\chi_\perp = \chi_\perp^{\text{ref}} \left| \frac{T'}{T_{eq}'} \right|^{\sigma - 1}
\]

(1)

where the prime refers to the derivative in the radial direction, \(T_{eq}\) is the initial equilibrium temperature and \(\sigma\) is the stiffness parameter. This model ensures that the equilibrium temperature gradient is consistent with the input power (\(H_{eq}\) in the pressure equation), and provides the desired behavior leading to a strong excitation of turbulent transport when the stiffness parameter \(\sigma\) is large. The level of heat transport exhibits a smooth transition at around \(\left| T_{crit}'/T_{eq}' \right| = 1 - 1/(\sigma - 1)\) between a low transport regime representative of collisional processes, and a high transport regime representative of turbulent ones (see figure 1).

This stiffness model can be applied to derive an evolution equation for the island width under the effect of a localized heat source at the O-point. For a large island, such that the temperature is completely flattened inside the island separatrix, the associated Rutherford equation is

\[
I_1 \tau_R \partial_t W = a \Delta' + a \Delta_{\Omega}'(P_{RF})
\]

(2)
with $W \equiv w/a$ the island width normalized to the minor radius $a$ of the plasma, $\tau_R = \mu_0 a^2 / \eta$ the resistive time, $I_1 \approx 0.82$, $\Delta'$ the tearing stability index, and [5]:

$$a \Delta'_f(p_{RF}) = -C_\Omega(\mu_c, \sigma) \frac{a q \mu_0 R \Omega}{J_s} \frac{P_{eq}}{N_{\chi_{rf}}^{ref} T_s} \left( \frac{P_{RF}}{P_{eq}} \right)^{1/\sigma}$$

$$C_\Omega(\mu_c, \sigma) \approx \frac{3}{4 \pi^2} \left[ 0.8 + \frac{0.6}{\sigma} - 1.09 \frac{\mu_c}{\sigma} + 0.24 \left( \frac{\mu_c}{\sigma} \right)^2 - 0.23 \frac{\mu_c}{\sigma} \ln \frac{\mu_c}{\sigma} \right]$$

where $J \approx rR$ is the Jacobian and $J_\Omega$ the ohmic current, $P_{RF}$ the total RF power injected at the O-point over the width $\delta_H$, $\mu_c = (\delta_H/W)^2$, $P_{eq}$ the power injected inside the resonant surface (in the absence of RF heating), $N$ the plasma density and $T_s$ the temperature at the resonance. In the absence of stiffness ($\sigma = 1$), this expression is consistent with previous results [6].

**Numerical simulations** Numerical experiments are performed with the non linear MHD code XTOR-2F [7] using the resistive MHD model:

$$(\partial_t + \mathbf{V} \cdot \nabla) \rho = -\rho \nabla \cdot \mathbf{V} - \nabla \cdot \mathbf{\Gamma}_{an} + \Sigma$$

$$(\partial_t + \mathbf{V} \cdot \nabla) \mathbf{p} = -\mathbf{\Gamma}_p \mathbf{V} + \mathbf{H}_{eq} + \mathbf{H}_{RF} - (\Gamma - 1) \nabla \cdot \mathbf{q}_\chi$$

$$\rho(\partial_t + \mathbf{V} \cdot \nabla) \mathbf{V} = \mathbf{J} \times \mathbf{B} - \nabla p + \nabla \cdot \mathbf{\nabla} \mathbf{V}$$

$$\partial_t \mathbf{B} = \nabla \times [\mathbf{V} \times \mathbf{B} - \eta (\mathbf{J} - \mathbf{J}_{CD})]$$

with $\rho$ the mass density, $p$ the total pressure, $\mathbf{V} = \mathbf{V}_E + \mathbf{V}_{\|i}$, $\mathbf{V}_E = \mathbf{E} \times \mathbf{B} / B^2$ and $\mathbf{V}_{\|i}$ the parallel ion velocity, $\Gamma = 5/3$, $H_{eq} \equiv -(\Gamma - 1) \mathbf{V} \cdot \mathbf{\chi}_{\perp} \nabla \cdot \mathbf{\chi}_{\perp} p_{eq}$ is the heat source ($p_{eq} \equiv p(t = 0)$), $H_{RF}$ is the RF heat source and $\mathbf{q}_\chi = -\rho \mathbf{\chi}_{\|} (\mathbf{b} \cdot \nabla T) - \rho \mathbf{\chi}_{\perp} \nabla \cdot T$ is the diffusive heat flux ($\mathbf{b} \equiv \mathbf{B} / B$), with $T = p/\rho$. The non-inductive current density source is $\mathbf{J}_{CD} = (\mathbf{J}_\phi - E_0/\eta)_{t=0}$ with $E_0$ a constant prescribed at the edge such that $E_0/(\eta(0) J_\phi(0)) = 1$, i.e. the current is fully inductive at the plasma center. The RF heat source is implemented as follows [8]:

$$\partial_t H_{RF} = \nu_f \left( H_{RF}^S - H_{RF}^H \right) - \nabla \cdot \mathbf{q}_{RF}^H$$

$$\mathbf{q}_{RF}^H = -\mathbf{\chi}_{\perp}^H \nabla H_{RF} - \left( \mathbf{\chi}_{\|}^H - \mathbf{\chi}_{\perp}^H \right) \mathbf{b} \cdot \nabla H_{RF}$$
where $\nu_f = \nu_{ei} (v_{th}/v_{res})^2$ is the collision frequency of the fast electrons, with $v_{th}/v_{res} = 1/2$ and a source term $H_{RF}^S$ defined as 1D or 3D Gaussian (with $y \equiv \sqrt{\psi}$):

$$H_{RF}^S(y, \theta, \phi) = H_0^S \exp \left( -\frac{(y-y_0)^2}{2\sigma_r^2} - \frac{(\theta-\theta_0)^2}{2\sigma_\theta^2} - \frac{(\phi-\phi_0)^2}{2\sigma_\phi^2} \right) \quad (11)$$

We apply a RF heat source at the O-point of a saturated island of helicity $(m = 2, n = 1)$ and of width $W = w/a \approx 8.4\%$ with $a$ the minor radius of the plasma (fig. 2). The island decay rate (see fig. 3) is measured 25ms after the heating is applied, and compared with the theoretical predictions of eq. (3) (see fig. 4), showing a good agreement [9]: at low $P_{RF}/P_{eq}$ the island is better stabilized in a stiff plasma, while the stabilization efficiency saturates at higher values, with a dependency $(P_{RF}/P_{eq})^{1/\sigma}$.

A plasma with stiff temperature profile is however very sensitive to residual heat sources in the island region, where they can produce a bump of temperature, as sometimes observed, and verified in the simulations. In such case, the stabilizing effect expressed in eq. (3) applies, where $P_{RF}$ is replaced by the power deposited inside the island $P_{isl}^eq = 8\pi w \int H eq$. The stabilization effect that also varies as $(P_{isl}^eq/P_{eq})^{1/\sigma}$ is large for $\sigma = 8$, and we verify that the island saturation is indeed reduced. But the control capability is also severely damped, due to the fact that the temperature gradient is already close to the turbulence threshold value before the RF application, as shown in figure 5. The island decay rate can then be fitted by making the replacement $(P_{RF}/P_{eq})^{1/\sigma} \to (P_{RF} + \alpha P_{isl}^eq)/P_{eq})^{1/\sigma} - (\alpha P_{isl}^eq/P_{eq})^{1/\sigma}$ in eq. (3), with $\alpha = 0.1$, giving a modified Rutherford equation:

$$I_1 \tau_R \partial_t W = a\Delta + a\Delta' (P_{RF} + \alpha P_{isl}^eq) - a\Delta'_\Omega (\alpha P_{isl}^eq) + a\Delta_\Omega (P_{isl}^eq) \quad (12)$$
**Conclusion** We have shown analytically and numerically that the stabilization efficiency of a localized heating at the O-point of a magnetic island is larger in a plasma with stiff temperature profile for relevant situations where $P_{RF}/P_{eq}$ is lower than unity (it is expected to be around 0.15 in ITER). The presence of residual heat sources in the island region lowers its saturation size, but also drastically limits the control capability of a pure RF heating. Implications for Neoclassical Tearing Mode control in ITER are discussed in [10].

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**References**


