Electron capture in the dense semiclassical plasma

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Introduction. Investigation of the interaction between particles and plasma properties is of great interest in many areas of physics such as atomic and plasma physics. Also, it is important for the development of the plasma technologies. One of the elementary processes in plasma is the electron capture process due to electron and atom collision. The process of the electron capture by an atom was investigated in many studies [1-5, 9]. In this paper electron capture by the hydrogen atom was considered. The neutral hydrogen atom can be transformed into the negative hydrogen ion due to polarization electron capture. The negative hydrogen ion plays an important role in partially ionized plasmas, also it is used for high energy accelerators and for the neutral beam injection systems of fusion devices. In work [5] the electron capture cross section was theoretically evaluated in the framework of the perturbation theory, where unperturbed linear trajectory of the projectile was considered.

In this work we used the interaction potential between the electron and the atom in partially ionized hydrogen plasmas, which was presented in works [6-8]. This effective potential, taking into account the quantum-mechanical effects of diffraction of particles and plasma screening effects, has finite values at the distance close to zero. It has the following form:

$$\Phi_{ea}(r) = -e^2\alpha \left( e^{-B \Phi} (1 + Br) - e^{-A \Phi} (1 + Ar) \right)^2 \left( 2r^4 \left( 1 - 4 \lambda_{ea}^2/r_p^2 \right) \right),$$

where

$$A^2 = \left( 1 + \sqrt{1 - 4 \lambda_{ea}^2/r_p^2} \right) \left( 2 \lambda_{ea}^2 \right), \quad B^2 = \left( 1 - \sqrt{1 - 4 \lambda_{ea}^2/r_p^2} \right) \left( 2 \lambda_{ea}^2 \right).$$

Here, $\lambda_{ea} = \hbar / (2\pi \mu_{ea} k_B T)^{1/2}$ is the de Broglie wavelength, $r_p = \sqrt{k_B T / (4 \pi n e^2)}$ is the Debye length, $k_B$ denotes the Boltzmann constant, $\mu_{ea} = m_e m_a / (m_e + m_a) \approx m_e$ is the reduced mass of the atom and electron pair, $\alpha$ is the polarizability of the atom. For hydrogen atom it equals $4.5a_B^3$, $a_B = \hbar ^2 / (m_e e^2)$ is the Bohr radius.

Theory and methods. The Bohr-Lindhard method has been applied to obtain the electron capture radius, capture time and electron capture probability. In the Bohr-Lindhard
method [2], it has been known that the electron capture happens when the distance between
the atom and moving electron is smaller than the electron capture radius $R_{cap}$. This electron
capture radius is determined by equating the kinetic energy of moving electron and the
interaction energy between the electron and the atom.

$$
\frac{e^2}{\alpha} \left( e^{-BR_{cap}} \left(1 + BR_{cap}\right) - e^{-AR_{cap}} \left(1 + AR_{cap}\right) \right)^2 \left(2R_{cap}^4 \left(1 - 4\frac{A^2}{r_d^2}\right)\right) = \frac{m v_p^2}{2},
$$

where $v_p$ is the velocity of moving electron, $m v_p^2 / 2$ is the kinetic energy of moving
electron. The interaction energy provided by the polarization interaction should be greater
than the kinetic energy of the moving electron in the frame of the hydrogen atom.

The time of the capture was determined as the time, when electron moves within capture
radius. It was found from the ratio of the traversed path of the electron to the velocity of the
electron.

$$
t_{cap} = \begin{cases} \frac{2}{v_p} \sqrt{R_{cap}^2 - b^2} & \text{for } b \leq R_{cap} \\ 0 & \text{for } b \geq R_{cap} \end{cases}.
$$

It should be noted that the electron is captured, if the impact parameter is less than the capture
radius $b < R_{cap}$. The impact parameter $b$ is the vertical distance between the centers of the
moving electron and the atom.

The electron capture probability is defined by the ratio of the collision time to the
electron orbital time:

$$
P_{cap} \ b = \tau^{-1} \int_{-t_{cap}}^{t_{cap}} dt ,
$$

Using the electron capture probability, the electron capture cross section was
calculated on the basis of the following expression:

$$
\sigma_{cap} = 2\pi \int db b P_{cap} (b) ,
$$

where $\tau = a_n / v_n$ is the electron orbital time, $a_n = n^2 a_n / Z$ is the $n$-th Bohr radius of the
hydrogenic atom, $Z$ is charge number of the atom, $v_n = Z c e / n$ is the electron velocity of the
$n$-th Bohr orbit.

In this work the pair collision of impacting electron with atom was considered. The
influence of other plasma particles is taken into account by the effective potential (1). The
equations of motion of the electron in the field of the motionless hydrogen atom were
numerically solved.

**Results.** Figure 1 and figure 2 shows the differential cross section of electron capture obtained on the basis of the perturbation theory (4-6). These figures represent the dependence of the electron capture cross section on the impact parameter for various values of the coupling parameter and density parameter. As one can see, the differential cross section significantly increases with grow of the temperature, and decreases in more dense plasma.

The trajectories of an electron near a hydrogen atom, calculated on the basis of the numerical solution of the equations of motion of electron with initial velocity and impact parameters $b$, are shown in fig. 3. In this figure, the hydrogen atom is indicated by a circle.

Comparison of the differential cross section obtained by the perturbation theory and by solving the equation of motion was presented in figure 4.

![Fig. 1. The differential cross section for different values of the coupling parameter.](image1)

![Fig. 2. The differential cross section for different values of the density parameter.](image2)

![Fig. 3. The trajectories of the electron near the atom calculated by the equation of motion, $\Gamma = 0.2$, $r_0 = 3$, $v_x^* = 1$, $v_y^* = 0$.](image3)

![Fig. 4. Comparison of the differential cross section.](image4)
Conclusion

In this paper process of electron capture by the hydrogen atom was investigated. The interaction between the electron and the atom takes into account the screening effect at large distances and the effect of diffraction at small distances.

The differential cross section was obtained on the basis of perturbation theory and the equation of motion. Results were compared. The differential cross section equals to zero at large values of the impact parameter and grows with an increases in the coupling parameter. These results provide useful information on electron capture process in partially ionized plasma.

Acknowledgments

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References