Do hydrodynamical models underestimate exchange effects? Comparison with kinetic theory for electrostatic waves

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In dense or cold plasmas with degenerate electrons, the quantum mechanical effect of electron exchange can be important [1]. Quantum hydrodynamical models can include exchange through a potential derived from time-independent functional theory (TIDFT) [2], but because of the time-independence, the accuracy for dynamical problems is unclear. To investigate this accuracy, we compare with a kinetic model that includes exchange effects [3, 4], for the case of electrostatic waves [5].

Concretely, we compute the exchange correction to the electrostatic electron susceptibility for all phase velocities, at $T = 0\,\mathrm{K}$. We find that for low phase velocities (ion acoustic waves), the susceptibility in the kinetic model is an order of magnitude larger than the hydrodynamical one. The large discrepancy is due to wave-particle interaction that is lost in fluid models. However, for phase velocities large compared to the Fermi velocity ($v_\phi \gtrsim 2.5 v_F$) the hydrodynamical and kinetic susceptibilities agree rather well.

Our results have implications for model choice: for dense and cold plasmas, in addition to exchange, particle dispersive effects can be important. Relative to classical terms, the contributions scale as $H^2 = \hbar^2\omega_p^2/m_e^2 v_F^4 \propto n_e^{1/3}$ in both cases, where ω_p, m_e, n_e are the electron plasma frequency, mass, and number density, respectively. Because the numerical coefficient for exchange can be large in the low-frequency regime, using a model which includes particle dispersion but not exchange effects cannot be justified. As a further consequence, a quantum mechanical treatment including exchange may be necessary for a modest value of $H^2 \sim 0.1$, i.e., even for electron densities somewhat below those of metals (for which $H^2 \sim 1$).

References

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