Weak drift wave turbulence and the statistics of random matrices

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A statistical analysis of the drift wave (weak) turbulence necessarily starts with the linear eigenmodes. A weak nonlinearity can be seen as a vertex of an interaction where the elementary propagators correspond to the set of orthogonal eigenfunction of the linear operator and a renormalization theory can be developed. The first nonlinearity is the interaction between the non-adiabatic part of the density response and the electrostatic perturbation, which requires two field calculation. We note, in the present work, the possibility of another technical approach which introduces the nonlinearity through a perturbation of the complex roots of the functions of the base.

For the drift wave in sheared magnetic field the eigenfunctions are Hermite polynomials.

With the order scaled by an artificial time parameter (which maintains the orthogonality) the Hermite polynomials verify an equation of diffusion with a negative coefficient of diffusion. By an inverse Hopf-Cole transformation one obtains the Burgers equation with the same negative viscosity [Blaizot&Nowak, Phys Rev E 82, 051115 (2010)]. Evolving in the artificial time, the solution of this equation exhibits a shock formation, which, due to the negative viscosity is accompanied by oscillations. There is a connection between this solution and the average resolvent of the hermitean matrix with Gaussian random entries.

Now we interpret the introduction of the weak nonlinearity of the turbulent drift waves as a broken orthogonality of the modified set of functions, which depart from the linear drift wave eigenmodes. Then the diffusion with a negative coefficient is modified by an averaged term which acts as a source. However we adopt the approximative procedure to modify directly the complex singularities that define the inverse Hopf-Cole transformed function. Since this is connected with the resolvent of the hermitean random matrix set, the modification is reflected in the density of the eigenvalues.

We discuss the possibility to use this technical approach in order to obtain renormalization of the drift wave propagator in weak turbulence.