On the interpretation of perturbative momentum transport studies

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In tokamaks, torque modulation from tangential neutral beam injection (NBI) is routinely used to investigate toroidal momentum transport. The momentum transport coefficients (diffusion χ_φ, convection V_φ and residual stress C_φ) are inferred from the response of the toroidal angular frequency ω_φ to the torque modulation. The experiments are usually designed to achieve a small modulation of the toroidal rotation (typically by $5-20\%$) so that the transport equations can be linearised around the stationary state. For instance, the diffusive term in the momentum flux is written as $\chi_\varphi \omega'_\varphi \sim \chi_{\varphi,0}\omega'_{\varphi,1} + \chi_{\varphi,1}\omega'_{\varphi,0}$, where ω'_φ is the radial derivative of the toroidal angular frequency, the "0" subscript refers to the stationary background and the "1" subscript to the time dependent perturbation. In practice, the modulation of the transport coefficients ($\chi_{\varphi,1}$ in the example above) is often neglected. This approximation is made to decrease the number of unknowns and obtain a simpler system of equations. However, there is no simple ordering argument to support this approximation, quite the opposite. Indeed, as NBI is not only a momentum source, but also an energy source, it is experimentally difficult to avoid modulating the temperature by some $2-5\%$. Simply assuming a Bohm or gyro-Bohm dependence, the $2-5\%$ temperature modulation would result in a momentum diffusivity modulation of the same order of magnitude, which cannot be neglected in the linearised momentum flux.

The purpose of the present work is to quantify the impact of a modulation of the turbulence intensity in perturbative transport studies, with a focus on momentum transport. To this end, the temporal response of the toroidal angular frequeny and ion temperature to a modulation of the torque and energy sources is computed numerically for given transport coefficients, based on the following simplified 1D transport equations:

$$n\frac{\partial \omega_\varphi}{\partial t} + \frac{1}{V'}\frac{\partial}{\partial r}\left[nV'(1+\alpha)\left(-\chi_\phi \frac{\partial \omega_\varphi}{\partial r} + V_\varphi \omega_\varphi + C_\varphi\right)\right] = \frac{\mathscr{S}_\varphi}{m_i R_0^2} \quad (1)$$

$$n\frac{\partial T_i}{\partial t} + \frac{1}{V'}\frac{\partial}{\partial r}\left[nV'(1+\alpha)\left(-\chi_i \frac{T_i}{\partial r} + V_i T_i\right)\right] = \mathscr{S}_i 1 \quad (2)$$

Here, n is the plasma density, r the minor radius, $V' = \partial V/\partial r$ the radial derivative of the plasma volume, $\mathscr{S}_\varphi$ the torque density, m_i the ion mass, R_0 the reference major radius, T_i the ion temperature, χ_i the ion heat diffusivity, V_i the ion heat pinch and $\mathscr{S}_i$ the ion power density. For simplicity, the (convective) particle flux contribution and the temporal variation of the density were neglected. The parameter α is used to introduce a modulation of the turbulence intensity. It is either taken to be zero (no modulation of the transport coefficients) or equal to $\alpha_{gB} = (T_{i,1}/T_{i,0})^{3/2}$ (gyro-Bohm dependence).

All cases under investigation are based on a plasma with $R_0 = 0.9$m, $a = 0.25$ m, an average NBI power and torque of $P_{\rm NBI} = 1$ MW and $T_{\rm NBI} = 0.2$ N.m, respectively, and a broad NBI source (full width at half maximum of $0.3a$) peaked at $r/a = 0.2$. The reference momentum transport coefficients are shown with a thick black line in Fig. 2. The NBI source is modulated at $f = 20$ Hz with a 50/50 duty cycle and a square waveform. The torque modulation amplitude is $\pm 10\%$ for all cases and the power modulation amplitude is scanned from $\pm 1\%$ to $\pm 10\%$. The resulting modulation of the toroidal angular frequency and ion temperature at $r/a = 0$ is shown in Table 1. As the temperature modulation gets larger, the subsequent modulation of the transport coefficients leads to a decrease of the toroidal rotation modulation. To better assess

$\mathscr{S}_i$ modulation	1%	2%	5%	10%	5%
α	$\alpha_{\rm gB}$	$\alpha_{\rm gB}$	$\alpha_{\rm gB}$	$\alpha_{\rm gB}$	0
$\delta T_i/T_i$	0.4%	0.8%	2%	4%	4.8%
$\delta\omega_\varphi/\omega_\varphi$	6.6%	6.3%	5.2%	3.3%	7.2%

Table 1: *Relative temperature and toroidal rotation modulation at $r/a = 0$.*

this effect, the Fourier transform of ω_φ is computed and the amplitude and phase profiles at $f = 20$ Hz are shown in Fig. 1. The amplitude and phase profiles are both strongly affected by the modulation of the transport coefficients. To quantify the impact of the amplitude and phase profile distortion, the transport coefficients are computed from the modulated data, assuming that there was no turbulence intensity modulation, and compared to their input values. This is achieved by solving the following system of equations obtained from the $f_0 = 0$ Hz and $f_1 = 20$ Hz Fourier components of Eq. (3), after linearization:

$$-\chi_\varphi \omega'_{\varphi,f_0} + V_\varphi \omega_{\varphi,f_0} + C_{\varphi,f_0} = g_{\varphi,f_0} \tag{3}$$

$$-\chi_\varphi \omega'_{\varphi,f_1} + V_\varphi \omega_{\varphi,f_1} = g_{\varphi,f_1} \tag{4}$$

with

$$g_\varphi = \frac{1}{V'}\int V'\left[\frac{\mathscr{S}_\varphi}{m_i R_0^2} - n\frac{\partial \omega_\varphi}{\partial t}\right] \mathrm{d}r \tag{5}$$

This system of equations is solved at each radial position using a matrix formulation as in [1] and the resulting transport coefficients shown in the top row of Fig. 2. The deviation of the re-

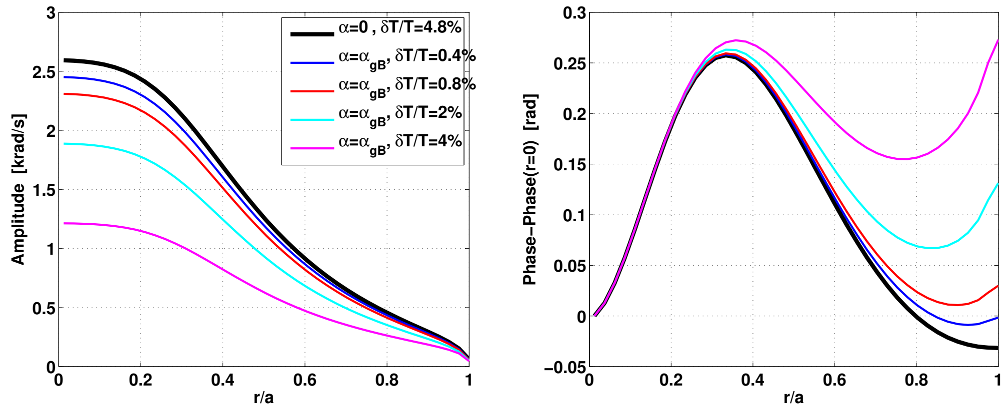

Figure 1: *Toroidal rotation frequency modulation amplitude (left plot) and phase (right plot) profiles at* $f = 20\,Hz$ *for cases without (black line) and with (other lines) modulation of the turbulence level. The temperature modulation achieved on axis is indicated in the legend.*

constructed momentum transport coefficients from their input value is quite large and increases with the amplitude of the temperature modulation. The observed discontinuities in the reconstructed coefficients coincide with regions where the determinant of the system of equations is zero or near zero. In these regions, any error on the toroidal rotation, momentum source or, in our case, on the assumptions is dramatically amplified [1]. Even when the temperature modulation is less than 1% the reconstruction is not satisfactory, which is a serious issue for perturbative momentum transport studies. In principle, the reconstruction could be improved by solving for the turbulence intensity modulation α in addition to the momentum transport coefficients. This,

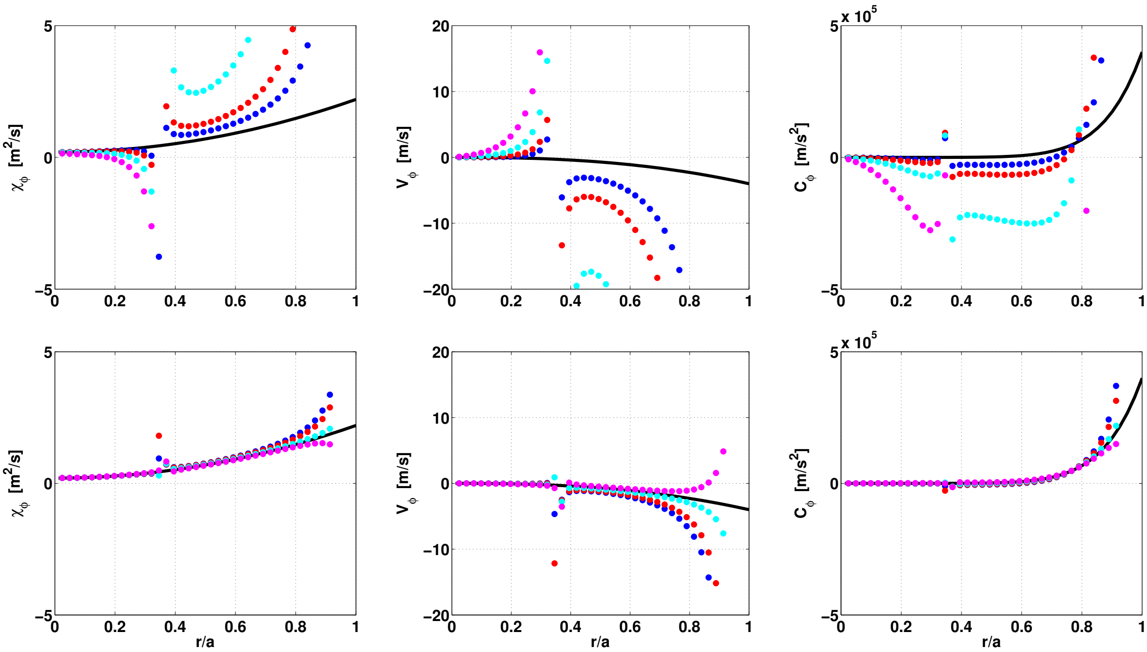

Figure 2: *Input (black lines) and reconstruced (color dots) momentum transport diffusivity* χ_φ *(left), pinch* V_φ *(middle) and residuals stress* C_φ *(right) coefficients. The modulation of the transport coefficients is neglected in the top row and taken into account in the bottom row.*

however, brings additional unknowns (amplitude and phase of α) and the system of equations needs to be extended. One possibility is to combine the $f_0 = 0\,\mathrm{Hz}$, $f_1 = 20\,\mathrm{Hz}$ and $f_2 = 20\,\mathrm{Hz}$ Fourier components of Eq. (3) and Eq. (4), after linearization:

$$-\chi_\varphi \omega'_{\varphi,f_0} + V_\varphi \omega_{\varphi,f_0} + C_{\varphi,f_0} = g_{\varphi,f_0} \tag{6}$$

$$g_{\varphi,f_0}\alpha_{f_1} - \chi_\varphi \omega'_{\varphi,f_1} + V_\varphi \omega_{\varphi,f_1} = g_{\varphi,f_1} \tag{7}$$

$$g_{\varphi,f_0}\alpha_{f_2} - \chi_\varphi \omega'_{\varphi,f_2} + V_\varphi \omega_{\varphi,f_2} = g_{\varphi,f_2} \tag{8}$$

$$-\chi_i T'_{i,f_0} + V_i T_{i,f_0} + C_{i,f_0} = h_{i,f_0} \tag{9}$$

$$h_{i,f_0}\alpha_{f_1} - \chi_i T'_{i,f_1} + V_i T_{i,f_1} = h_{i,f_1} \tag{10}$$

$$h_{i,f_0}\alpha_{f_2} - \chi_i T'_{i,f_2} + V_i T_{i,f_2} = h_{i,f_2} \tag{11}$$

where

$$h_i = \frac{1}{V'} \int V' \left[\mathscr{S}_i - n\frac{\partial T_i}{\partial t} \right] \mathrm{d}r \tag{12}$$

The reconstructed momentum transport coefficients obtained by solving the extended system of equations is shown in the bottom row of Fig. 2. The quality of the reconstruction is considerably improved and one also get access to the turbulence modulation level and to the heat transport coefficients. In contrast to the previous case, the reconstruction is better when the temperature modulation is large. This is because one now also relies on the heat equation: if the temperature modulation is vanishingly small, the determinant of the system of equations will tend towards zero and the solution will become increasingly sensitive to small errors.

This basic study reveals that a small modulation of the turbulence intensity can have a dramatic impact on the reconstruction of the transport coefficients. An accurate reconstruction can, however, still be obtained provided this modulation is taken into account. This requires an extension of the system of equations, for instance by adding the ion heat transport equation. The method has been validated on synthetic data assuming perfect measurements. Increasing the number of equations may, however, increase the sensitivity to the measurements uncertainty. Before applying the method on real data, it therefore remains to test the impact of the measurements uncertainty, sampling rate and integration time on the transport coefficients reconstruction.

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