Threshold power of amplitude modulated laser beam in complex plasma

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Abstract

A theoretical investigation describing the effect of dust on the threshold power of the amplitude modulated laser beam propagating in complex plasma has been done. In this analysis, momentum and energy balance equations have been solved simultaneously to govern the relation between the beam width parameter and normalizes length of progression of amplitude modified beam advancing in complex plasma. The dependence of beam width parameter on the progression length have been evaluated for different values of dust grain size and dust charge state. Moreover, the dependence of critical power of self-focusing on dust grain size and dust charge state have also been investigated. It is found that the critical power augments with the increase in dust grain size as well as with the increase of the dust charge state.

Introduction

Numerous articles have been come out with the self-trapping of an electromagnetic wave in complex plasma (consist of electrons, ions and positively or negatively charged dust grains) being supported by several reviews.\textsuperscript{1,2} Sometime, these small charged dust grains are responsible for reducing the attainment of industrial devices.\textsuperscript{3,4} whereas some mechanization preferred their presence.\textsuperscript{5} In addition, the presence of dust grains strongly influence the discharge properties of plasma.\textsuperscript{6-8} Dust grains become charged on account of accumulation of electrons on its surface, secondary electron emission, photoemission etc. and cause the variation in electron temperature and electron density. Generally, electron concentration decreases with increase in size of dust grain which leads to variation in critical power of self-focusing. For regulating the complex plasma properties, understanding of networking between complex plasma domains and dust grain parameters is essential. In this paper, we describe the non-linear self-focusing of beam in dusty plasma due to interaction between electrons and the dust grains by considering the non-linear heating of electrons.

In general, curves have drawn to show the dependence of beam width parameter on the normalized length of progression for the distinct parameters of amplitude modified beam
advancing in complex plasma and to analyse the change in critical power of self-focusing with the formation and charging of the dust grain.

**Dynamics of complex plasma**

Let \( n_{0i}, n_{0e}, \) and \( n_{0d} \) are the density of complex plasma constituents i.e., ions, electrons and negatively charged dust grains, respectively. In equilibrium, \( Z_{0d}e n_{0d} + e n_{0e} = Z_{0i}e n_{0i} \), where \(-e\) is charge on electron, \(-Z_{0d}e\) is the charge on dust and \(Z_{0i}e\) is the charge on ion. Let us assume a high power amplitude modified electromagnetic beam travelling along \( z \)-direction in a complex plasma with the electric vector given by

\[
\mathbf{E} = A_0 (r,z)(1+\mu \cos\omega t)\exp[-i(\omega t-kz)],
\]

where

\[
A_0^2 = A_{00}^2 (1+\mu_0 \cos\omega t)^2 e^{-r^2/r_0^2},
\]

\( k \) is the propagation constant, \( \omega \) is modulation frequency, \( \mu_0 \) is modulation index, \( A_{00} \) is amplitude of electric vector, \( r_0 \) is the initial width of the beam.

For \( z > 0 \),

\[
A_0^2 = \frac{A_{00}^2}{f^2} (1+\mu \cos\omega t)^2 e^{-r^2/r_0^2 f^2},
\]

where \( f \) represents the beam width parameter, \( r_0 \) is the beam spot size. The momentum and energy balance of the equation is given by

\[
m_e \frac{\overrightarrow{v_e} - \overrightarrow{v_c}}{dt} + m_e \overrightarrow{v_c} = -e \overrightarrow{E},
\]

and

\[
\frac{3}{2} K_b \frac{dT_e}{dt} + \frac{3}{2} K_b \delta_{dusty} v(T_e-T_0) = -\frac{1}{2} e \overrightarrow{E} \cdot \overrightarrow{v},
\]

where \( m_e, T_e, \) and \( v_e \) are the mass, temperature and velocity of electrons, respectively. \( K_b \) is the Boltzmann’s constant. \( \delta_{dusty} \) is a part of energy transfer via charging collision of electrons with dust, elastic collisions of electron with dust and ions, represented as

\[
\delta_{dusty} (= v_{\text{chg}} / v_{\text{chg}} + v_{\text{ei}} + v_{\text{ed}}) \quad \text{and} \quad v (= v_{\text{chg}} + v_{\text{ei}} + v_{\text{ed}}) \]

is the effective collision frequency, where \( v_{\text{chg}}, v_{\text{ei}}, v_{\text{ed}} \) are the charging, electrons-ions and electrons-dust grains collision frequencies respectively. The Eqs. (1) - (5) solved simultaneously to obtain dielectric constant in the existence of dust particles which is substituted in wave equation for electric field vector.
of propagating electromagnetic beam to obtain an equation to see the change in beam width parameter with the normalized length of progression, expressed as

\[
\frac{d^2f}{d\xi^2} = -\frac{1}{R_n^2} \frac{1}{f^3} (1 + \mu \cos \omega t)^2 + \frac{1}{R_n^2 f^3},
\]

where \( R_d = \frac{\omega_0^2}{c} \varepsilon_0^{-1/2} \varepsilon_0^2 \), \( R_n^2 = r_0^2 \varepsilon_0^2 / \varepsilon^2 A_{00}^2 \), \( R_n \) is the typical length of self-focusing and \( \varepsilon_2 = \left( \frac{\omega_p^2}{\omega_0^2} \right) \left( \frac{e^2}{6m_c \delta_{\text{dusty}}} T_0 \omega_0^2 \right) \) is an effective dielectric constant in complex plasma. In the right-hand side, the first part of Eq. (6) represent the nonlinear self-focusing term while the second diffraction divergence of the beam.

**Threshold power of self-focusing**

From Eq. (6), the critical power of self-focusing (for which \( \frac{d^2f}{d\xi^2} = 0 \), \( f = 1 \), for every values of \( \xi \)) can be written as

\[
P_{\text{cr}} = \frac{1}{8} \frac{1}{c^3} \frac{e_0^2}{\omega_0^2} \varepsilon_2.
\]

**Results and discussion**

Fig.1 (a) and Fig.1 (b) represent the deviation of beam width parameter with the normalized length of progression, \( \xi = (z/R_d) \) for different parameters of dust grain size and dust charge number and Fig.2(a) and Fig.2(b) represent the rate of change of critical power of self-focusing with the dust grain size and dust charge number in the following domain: \( \omega_p / \omega_0 = 31.167, \delta_{\text{dusty}} = 2.03 \times 10^{-4}, T_0 = 6.4 \text{ eV}, Z_d \) (dust charge state) = -2 \times 10^4 and \( A_d \) (dust grain size) = 10^{-4} cm, \( A_{00} = 10^5 \text{ stat volt cm}^{-1}, e_0 = 0.97, \varepsilon_2 = 0.8 \times 10^{-10}, e = 4.8 \times 10^{-10} \text{ esu}. \) With these parameters, critical power comes out to be 0.947\times10^{18} \text{ ergs/s}. Fig.1(a) and Fig.1(b) show that the self-focusing of the amplitude modulated beam increases with the increases in dust grain size because of dominating self-focusing term whereas decreases with the enhanced dust charge number due to strengthening in diffraction divergence term. Fig.3 and Fig.4 show that the critical power of self-focusing augmented with the increase in dust grain size as well as with the increase of the dust charge state. This is because with the increase of dust grain size and dust charge state, more and more accumulation of electrons take place on its surface which results in decrease of electron density and increase of critical power of self-focusing.

**References**


**Figure Caption**

Fig.1(a): Dependence of \( f \) on \( \zeta \), for different values of dust grain size labelled as a1, a2, a3 and a4 corresponds to dust radius, \( r_d = 1 \mu m, 5 \mu m, 10 \mu m \) and \( 15 \mu m \), respectively.

Fig.1(b): Dependence of \( f \) on \( \zeta \), for different values of dust charge number labelled as \( z_1, z_2, z_3 \) and \( z_4 \) corresponds \( Z_d = -8 \times 10^3, -6 \times 10^3, -4 \times 10^3, -2 \times 10^3 \), respectively.

Fig.2(a): Variation of critical power \( P_{cr} \) (in ergs/s) with dust grain radius \( r_d \).

Fig.2(b): Variation of critical power \( P_{cr} \) (in ergs/s) with dust charge state \( Z_d \).