Instability of Electron-Positron Plasmas Driven by Pressure Gradients

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Recent advances in laboratory electron-positron experiments have brought about the possibility of directly studying collective effects in pair plasmas [1, 2, 3]. In addition to validating theory predictions, this opens up the possibility of closely examining processes thought to impact the dynamics and evolution of Gamma Ray Bursts [4], among other astrophysical scenarios.

Here, the focus shall lie on a new, kinetic plasma instability [5], which constitutes an interesting candidate for producing fluctuations and turbulence driven by density and temperature gradient. A recent study of turbulence in helium plasmas [6] in the Large Plasma Device [7] has demonstrated that gyrokinetic [8] modeling is able to predict important observation signatures in the steep-pressure-gradient region of the machine. In particular, a strong magnetic fluctuation component $B_\parallel$ parallel to the guide field arises from the Gradient-driven Drift Coupling (GDC), which constitutes the primary turbulence drive in these regions.

The mechanism of the GDC instability [5] operates as follows. In an orthogonal coordinate system – spanned by the $z$ direction parallel to the guide field, the $x$ direction along which a gradient in the background density and/or temperature exists, and the $y$ direction – consider a perturbation of the electrostatic potential $\Phi$ at a single wavenumber $k_y$. Such a perturbation will cause an $E \times B$ drift along $x$, advecting denser/hotter material from up-gradient and more dilute/colder material from down-gradient. Local force balance turns this pressure fluctuation into a $B_\parallel$ fluctuation; the latter produces its own drift $\nabla B_\parallel \times B$, which is indirectly proportional to a particle species’ charge, and describing which analytically or numerically requires kinetic treatment. As a quasi-neutral plasma will undergo local charge separation due to the $\nabla B_\parallel \times B$ drift, the original $\Phi$ perturbation is reinforced, leading to instability. A second, stable mode branch exists, with the same absolute value growth rate.

This mechanism works independently of the particle species mass ratio and thus applies to electron-positron pair plasmas, as well. While Ref. [9] conclusively demonstrates the absence of any pressure-gradient-driven electron-positron plasma instabilities in an unsheared slab when considering $\Phi$ and $B_\perp$ fluctuations, this argument no longer holds when retaining $B_\parallel$. Even at very low values of the normalized electron pressure $\beta$, GDC growth is found in pair plasmas. As detailed in Ref. [10] and focusing on the strong-guide-field limit as well as scales much larger than the gyroradius, one may derive the GDC growth rate $\gamma$ in an electron-positron plasma, assuming identical temperature profiles for both species and focusing on the dominant mode at
\( k_x = k_z = 0, \)
\[
\gamma = \frac{\sqrt{2}(\omega_n + \omega_T)}{\sqrt{(2 + 1/\beta)(2 + \lambda_D^2)}},
\]
where \( \omega_n = -(L_z/n_0)dn_0/dx \) and \( \omega_T = -(L_z/T_0)dT_0/dx \) are the gradients of the background density \( n_0 \) and temperature \( T_0 \) normalized to a macroscopic length scale \( L_z \), and the Debye length measured in gyroradii is denoted by \( \lambda_D \). The growth rate itself is normalized to units of thermal velocity to \( L_z \).

Reference [10] demonstrates excellent agreement between this expression and direct simulations, in addition to exploring the case of finite \( k_z \). Fundamentally, the inclusion of parallel dynamics couples the \( B_{\perp} \) drift into the aforementioned \( \Phi \) and \( B_{\parallel} \) drift system. This addition partially mitigates the stabilization effect of \( k_z \), but complete GDC stabilization is still found to occur at \( k_z L_z \ll \omega_{n,T} \).

A similar effect may be produced by introducing finite background magnetic shear into the otherwise homogeneous guide field topology. The normalized shear is defined as \( \hat{s} = L_z/L_s \), with the shear scale length \( L_s \). Figure 1 shows the stabilizing effect exerted by finite shear on the GDC growth rate. Note that \( \hat{s} \neq 0 \) introduces a quasi-periodicity in the parallel boundary condition, coupling different \( k_x \) as well as mixing parallel wavenumbers \( k_z \); both effects by themselves cause GDC stabilization. Unsurprisingly, the GDC growth rate nears zero at \( \hat{s} = L_z/L_s \sim 0.3 \ll 100 = \omega_{n,T} \), consistent with the above \( k_z \neq 0 \) considerations. Note that one may easily verify that the mode remains a GDC even at finite \( \hat{s} \) by artificially removing \( B_{\parallel} \) from the simulations: in this case, no growth is observed throughout the \( \hat{s} \) range shown here.

This finding explains why no unstable GDC has so far been identified in magnetic confinement fusion experiments, where magnetic shear scale lengths tend to be only about one order of magnitude larger than gradient scale lengths, or \( \hat{s} \geq 0.1 \omega_{n,T} \). In this context, note that a linearly stable GDC may be excited nonlinearly and may be related to the observation of correlated density and parallel magnetic field fluctuations in the Madison Symmetric Torus at high frequencies in the range of 50 – 100 kHz [11].
However, low-shear configurations remain promising candidates, assuming that other geometric properties do not interfere with the GDC mechanism and that toroidal instabilities do not dominate the linear instability picture to a degree where isolating GDC growth becomes infeasible.

Returning the focus to electron-positron pair plasmas, one may consider the implications of Eq. (1) on various physical systems, following again Ref. [10]. Laser-induced pair plasmas [1] achieve large $\beta \gg 1$ at small Debye lengths $\lambda_D \ll 1$, effectively reducing the GDC growth rate to the sum of the normalized gradients. Resulting growth times tend to be many orders of magnitude faster than plasma life times, supporting the notion that GDC-induced fluctuation may be detectable in present-day experiments.

A rather different set of physical parameters presents itself in the case of the APEX experiment [2, 3], which espouses low $\beta$ and large $\lambda_D$. This drastically reduces GDC growth rates and makes it likely that if any other instability – possibly arising from the dipole geometry of the device – were to appear, it would dominate over the GDC. If the GDC should remain the only instability in the system, however, its growth times are comparable with predicted plasma life times.

Lastly, Gamma Ray Bursts [4] are highly energetic astrophysical objects with very disparate pressure gradient scale lengths. Resulting GDC growth times vary from microseconds to years, suggesting that while not a global Gamma Ray Burst mechanism, GDC activity may play a role in some regions, particularly those with steep gradients.

References