Unified transport scaling laws for plasma blobs and depletions

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Introduction

Blobs or filaments are magnetic field aligned structures appearing in the scrape-off layers of magnetically confined plasmas. There they are responsible for a significant amount of radial particle and heat transport. Depletions on the other hand can be thought of as blobs with negative relative amplitude $\triangle n/n_0 < 0$, where n_0 is the density of the background plasma. Both blobs and depletions are generated near or close to the last closed flux surface. However, blobs and depletions propagate in opposite direction. On the other side in the night-side F-layer ionosphere plasma depletions appear as a result of the vanishing ionisation by solar radiation. These "bubbles" rise in the ambient plasma and can trigger turbulence in otherwise stable regions. This leads to the so-called equatorial spread-F phenomenon. In this contribution we present a unified model for the essential dynamics of both blobs and depletions. It is based on the combination of various scaling laws, analytical results and results from simulations.

Phenomenological 1d model "bubbles/stones"

The basic idea is to reduce the essential centre of mass dynamics to a one-dimensional model. The blobs and depletions are modeled as infinitely long cylinders with density $n_0 + \triangle n$ and radius ℓ immersed in an ambient plasma with density n_0. This is shown in Fig. 1. The coordinate system is aligned to the (effective) gravity g. This can either be the real gravity in the ionosphere or the effective gravity $g = C_s^2/R_0 = t_{e0}/m_iR_0$ in magnetically confined plasmas. We propose the following one-dimensional dynamical equation for the centre of mass velocity V

$$M_i \frac{dV}{dt} = F_g - F_b - c_1 V - \mathrm{sgn}(V) c_2 V^2 \qquad (1)$$

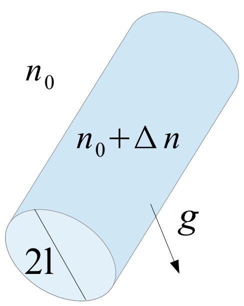


Figure 1: *"Top hat" density structure*

Here, the inertial mass appears as $M_i = (2/\mathcal{Q})\pi\ell^2(n_0 + 2\triangle n/9)$. The first term on the right hand side is the gravitational force $F_g = \pi\ell^2(n_0 + \triangle n)g$, which is just the mass of the filament times the gravity. The second term is the buoyancy $F_b = \pi\ell^2 n_0 g$, which includes the mass of the displaced ambient plasma. We introduce a linear friction term with $c_1 V = (2/\mathcal{Q})\pi\ell^2 n_0 g V/C_\mathrm{s}$. This term was derived recently in References [1, 2] from energetic arguments in seeded blob simulations. The last term is the nonlinear friction $c_2 V^2 = (1/\mathcal{R}^2)\pi\ell n_0 V^2$. In the model two fit parameters appear, which we determined to $\mathcal{Q} = 0.34$ and $\mathcal{R} = 0.85$.

Maximum Velocity

From Eq. (1) the maximum velocity of blobs and depletions is retrieved by setting $\mathrm{d}V/\mathrm{d}t = 0$. We get

$$\frac{\max|V|}{C_\mathrm{s}} = \left(\frac{\mathcal{R}^2}{\mathcal{Q}}\right)\frac{\ell}{R_0}\left(\left(1+\left(\frac{\mathcal{Q}}{\mathcal{R}}\right)^2\frac{|\triangle n|/n_0}{\ell/R_0}\right)^{1/2} - 1\right) \tag{2}$$

which for

$$\left(\frac{\triangle n/n_0}{\ell/R_0}\right)_\mathrm{c} \ll \left(\frac{2\mathcal{R}}{\mathcal{Q}}\right)^2 \tag{3}$$

reduces to

$$\frac{\max|V|_\mathrm{linear}}{C_\mathrm{s}} = \frac{\mathcal{Q}}{2}\frac{|\triangle n|}{n_0} \quad \text{linear scaling} \tag{4}$$

In the opposite limit we have

$$\frac{\max|V|_\mathrm{sqrt}}{C_\mathrm{s}} = \mathcal{R}\left(\frac{\ell}{R_0}\frac{|\triangle n|}{n_0}\right)^{1/2} \quad \text{square root scaling} \tag{5}$$

This limit can also be understood as incompressible limit since it is obtained from Eq (1) for $C_\mathrm{s} \to \infty$. In fluid models this scaling appears neglecting the compression of the $\boldsymbol{E}\times\boldsymbol{B}$ velocity. Equation (2) and (5) fit very well to numerical simulations as we show in Fig. 2.

Initial acceleration and Boussinesq approximation

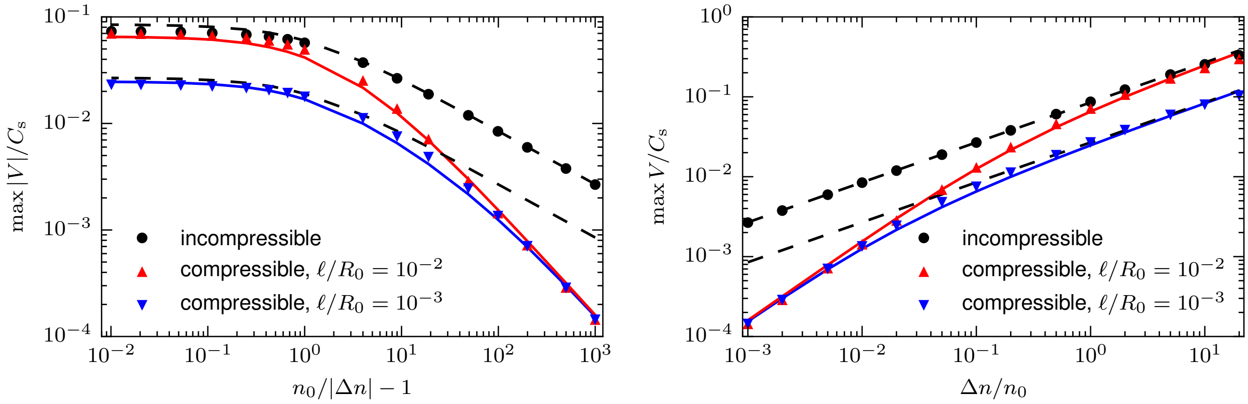
If in Eq. (1) we set $V = 0$ we are left with the initial acceleration of a seeded blob/depletion

$$\frac{A_0}{g} = \mathcal{Q}\frac{2S(0)}{M} \approx \frac{\mathcal{Q}}{2}\frac{\triangle n}{n_0 + 2\triangle n/9} \underset{\text{Boussinesq}}{\rightarrow} \frac{\mathcal{Q}}{2}\frac{\triangle n}{n_0}. \tag{6}$$

Equation (6) fits very well to numerical simulations as we show in Fig. 3 Interestingly, the Boussinesq approximation appears in the acceleration as a neglect of the changed inertial mass in Eq. (1) replacing it with the mass of the ambient plasma.

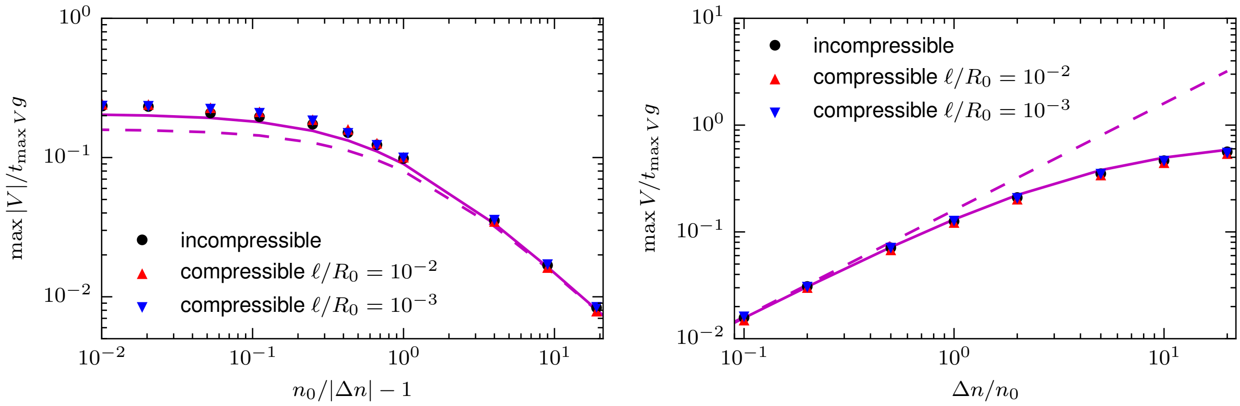
Current efforts and open issues

We are currently trying to include the recent results on temperature dynamics [3] in our models. To this end the temperature appearing in the sound speed C_s has to be modified. We assume



(a) depletions (b) blobs

Figure 2: *The maximum radial COM velocities of depletions(a) and blobs(b) for compressible and incompressible flows are shown. The continuous lines show Eq.* (2) *while the dashed line shows the square root scaling Eq.* (5) *with* $\mathscr{Q} = 0.32$ *and* $\mathscr{R} = 0.85$. *Note that small amplitudes are on the right and amplitudes close to unity are on the left side in plot (a)*



(a) depletions (b) blobs

Figure 3: *Average acceleration of depletions(a) and blobs(b) for compressible and incompressible flows are shown. The continuous line shows the acceleration in Eq.* (6) *with* $\mathscr{Q} = 0.32$ *while the dashed line is a linear reference line, which corresponds to the Boussinesq approximation.*

$C_s^2 = (t_{e0} + t_{i0} + \triangle t_e + \triangle t_i)/m_i$ in the gravitational force $\mathscr{F}_g$. Here, we take the temperature inside the blob/depletion. For the buoyancy and linear friction forces we take $C_{s0}^2 = (t_{e0} + t_{i0})/m_i$, which corresponds to the temperature of the ambient plasma. Inserted into Eq. (1) this leads to changed formulas for acceleration and the centre of mass velocities.

$$A_0 = \frac{\mathscr{Q}}{2} \frac{\triangle p_e + \triangle p_i}{m_i R_0 (n_0 + 2\triangle n/9)} \tag{7}$$

$$\max |V|_{\text{sqrt}} = \mathscr{R} \left(\frac{\ell |\triangle p_e + \triangle p_i|}{n_0 m_i R_0} \right)^{1/2} \tag{8}$$

$$\frac{\max |V|_{\text{linear}}}{C_{s0}} = \frac{\mathscr{Q}}{2} \frac{|\triangle p_e + \triangle p_i|}{p_{e0} + p_{i0}} \tag{9}$$

where $p_{e,i} = (n_0 + \triangle n)(t_{e,i} + \triangle t_{e,i})$, $p_{e,i0} = n_0 t_{e,i}$ and $\triangle p_{e,i} = p_{e,i} - p_{e,i0}$. The square root regime and the acceleration reproduce the results in Reference [3]. It remains to be seen if the linear regime can also be found in models with temperature dynamics but initial results look promising.

A remaining issue is of course how and if the parallel dynamics or sheath connected blobs can be included in such a simplified model.

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