Neoclassical impurity transport in stellarators

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Introduction: Heavy impurities originating from plasma facing components are typically not fully ionised in the core of fusion plasmas. Their central accumulation must be avoided, as power balance could not be maintained in the presence of the strong resulting radiation losses. Neoclassical *temperature screening* is the outward drive of the flux of impurity ions due to the main ion temperature gradient, which is known to occur in a tokamak. Thought to be absent in stellarators, we have shown [1] that it can appear in mixed collisionality plasmas, that is a plasma in which a heavy, highly charged and thus collisional, impurity is present in a low collisionality bulk plasma, where the main ions are in either of the $1/\nu$ or $\sqrt{\nu}$ regimes. Stellarator transport is not intrinsically ambipolar. An inward radial electric field is expected to develop in hot, low collisionality plasmas, and drive a strong inward flux of highly charged impurities [2]. This drive was found to vanish in the mixed collisionality plasma when the bulk ions are in the $1/\nu$ regime, and to be potentially weak in hotter plasmas where the bulk ions are in the $\sqrt{\nu}$ regime, particularly if the magnetic field geometry is well-optimised.

Formulation: The neoclassical radial flux of a heavy species, with mass m_z, density n_z and charge Ze, across nested magnetic flux surfaces in a mixed collisionality plasma is predominantly driven by parallel friction against the bulk ions (denoted by subscript i, charge e), $\Gamma_z = \left\langle f_z(\mathbf{v}_{dz} \cdot \nabla r) d^3v \right\rangle = \left\langle uBR_{zi\|} \right\rangle / Ze$, when the following condition on collisionality is satisfied: $1 \ll (n_i/\sqrt{Z} n_z)\nu_{*zz}\nu_{*iz}$. Here r is an arbitrary radial coordinate labelling flux surfaces, angled brackets denote a flux surface average, the collisionality $\nu_{*ab} = \nu_{ab}/\omega_{ta}$, where ν_{ab} is the characteristic collision frequency between species a and b, ω_{ta} is the characteristic transit frequency along the magnetic field and $\mathbf{v}_{dz}$ is the magnetic drift of the impurities. The equilibrium function u satisfies $\mathbf{b} \cdot \nabla u = -\mathbf{b} \times \nabla r \cdot \nabla (B^{-2})$, with the integration constant set such that $u = 0$ where $B = B_{max}$, the maximum field strength on the surface. The plasma confinement is assumed to be sufficiently good for the ion species' temperatures have equalised, $T_i = T_z = T$.

The friction is given by the linearised collision operator $R_{zi\|} = -R_{iz\|} = -m_i \int v_\| C_{iz} d^3v$. This can be determined analytically, as the disparate masses of the ions allow us to approximate the linearised collision operator with the momentum restoring form $C_{iz}(f_i) = \nu_D^{iz}(v)\left(\mathscr{L}(f_i) + 2v_{Ti}^{-2} v_\| V_{z\|} f_{Mi}\right)$, where $\mathscr{L}$ is the pitch angle scattering operator, the thermal speed is $v_{Ti} =$

$\sqrt{2T/m_i}$, the deflection frequency is $\nu_D^{iz}(v) = 3\pi^{1/2}/4\tau_{iz}x_i^3 = \hat{\nu}_D^{iz}/x_i^3$, where the normalised velocity $x_i = v/v_{Ti}$ and the collision time $\tau_{iz} = 3(2\pi)^{3/2}\sqrt{m_i}T^{3/2}\varepsilon_0^2/n_z Z^2 e^4 \ln\Lambda$. Incompressibility of the leading order flow follows from the drift kinetic equation for each species, so the parallel flow appearing here takes the general form $n_z V_{z\|} = (p_z/Ze)A_{1z}uB + K_z(r)B$, where the radial driving term $A_{1z} = d\ln p_z/dr + (Ze/T)d\Phi/dr$, with Φ the electrostatic potential. The flux function $K_z(r)$ is constrained by parallel momentum balance at zeroth order in the high collisionality expansion of the impurity drift kinetic equation, which sets $\left\langle BR_{zi\|}\right\rangle = 0$.

The piece of the bulk ion distribution function which is odd, f_i^-, with respect to the parallel velocity is needed to evaluate the parallel friction. A formalism by which this can be determined throughout the $1/\nu$ and $\sqrt{\nu}$ regimes was recently presented for a pure plasma in [3], and can be readily extended to account for the presence of an impurity. The bulk ion drift kinetic equation is split into pieces which are even and odd in $v_\|$, and the line integral of the even equation formally gives $f_i^-(r,\alpha,l,\varepsilon,\mu,\sigma) = \int_{l_0}^{l}\left[C_i^+(f_i) - \mathbf{v}_{di}\cdot\nabla f_i^+\right]dl'/v_\| + X(r,\alpha,\varepsilon,\mu,\sigma)$, where l is the length along a field line, α labels field lines on a surface, and the velocity space coordinates are the energy ε, magnetic moment μ and $\sigma = v_\|/|v_\||$. When the bulk ions are in the $1/\nu$ regime, the leading order even piece of the distribution is maintained near Maxwellian by collisions, $f^+ \approx F_{Mi}$. This is not the case in the lower collisionality $\sqrt{\nu}$ regime, where drift orbits can generate loss regions and the plasma is not generally in local thermodynamic equilibrium. However, it is known that confinement can be restored in the presence of a sufficiently strong radial electric field, creating a drift which can compete with the magnetic drift and keep the bounce-averaged orbits close to a flux surface, or when the orbit averaged magnetic drift is made small compared to the local value, by optimisation of the magnetic field. It is assumed that one of these conditions holds, so Φ is a flux function, $f_i^+ \approx F_{Mi} + F_1$, where $F_1 \ll F_{Mi}$ and the effect of F_1 principally appears in the trapped region of velocity space.

The odd piece of the distribution must vanish at a bounce point, therefore l_0 is set to be such a point in the trapped region of velocity space and l_{max}, where $B(l_{max}) = B_{max}$, otherwise. The integration constant X then goes to 0 in the trapped region, and is set in the passing region by the constraint resulting from the orbit average of the odd piece of the drift kinetic equation, $\left\langle BC_i^-(f_i)/v_\|\right\rangle = 0$. To evaluate this explicitly we need a form for the bulk ion self-collision operator. We take it to have a momentum restoring form analogous to C_{iz}, but allow for the full energy dependent deflection frequency $\nu_D^{ii}(v) = \hat{\nu}_D^{ii}\left[\phi(x_i) - G(x_i)\right]/x_i^3$, where $\phi(x)$ is the error function and $G(x)$ the Chandrasekhar function. The momentum restoring coefficient $\mathscr{V}_{i\|}$ appearing in place of $V_{z\|}$ is set by requiring momentum conservation in self-collisions: $\int v_\| C_{ii}(f_i)d^3v = 0$.

Results: With the model introduced above, the friction takes the form $R_{zi\|} = G_1(r)uB + G_2(r)sB + G_3(r)B$, where the flux functions $\mathscr{V}_{i\|}$ and K_z only appear in G_3. Applying the constraint $\langle BR_{zi\|}\rangle = G_1(r)\langle uB^2\rangle + G_2(r)\langle sB^2\rangle + G_3(r)\langle B^2\rangle = 0$, we see that we may eliminate G_3, and thus do not need to evaluate $\mathscr{V}_{i\|}$ or K_z explicitly. The radial impurity flux is then given by

$$\begin{aligned}
\Gamma_z = \frac{1}{Ze}\langle uBR_{zi\|}\rangle &= -\frac{m_i p_i}{Ze^2\tau_{iz}}\left[\frac{1}{Z}A_{1z}\left(\langle u^2B^2\rangle - \frac{\langle uB^2\rangle^2}{\langle B^2\rangle}\right)\right. \\
&\quad \left. -\left(A_{1i} - \frac{3}{2}A_{2i}\right)\left(\langle u\,(u+s)\,B^2\rangle - \langle (u+s)\,B^2\rangle\frac{\langle uB^2\rangle}{\langle B^2\rangle}\right)\right], \\
&\equiv n_z\left(D_{11}^{zi}A_{1i} + D_{11}^{zz}A_{1z} + D_{12}^{z}A_{2i}\right), \qquad (1)
\end{aligned}$$

where the driving term $A_{2i} = d\ln T/dr$. The term s, originating in the trapped particle drift, is zero in the $1/\nu$ regime and with the velocity space coordinates $\lambda = \mu/\varepsilon$, $\xi = \pm\sqrt{1-\lambda B}$, setting $\Phi = 0$ on the surface of interest, it is given in the $\sqrt{\nu}$ regime by

$$s(l) = \frac{3}{2}\int_{l_{max}}^{l} dl' \int_{1/B_{max}}^{1/B(l')} \frac{d\lambda}{\xi(l')}\overline{\xi\,(\mathbf{b}\times\nabla r)\cdot\nabla\left(\frac{\xi}{B}\right)}, \qquad (2)$$

where the overbar denotes an orbit average.

The impurity flux can be characterised in terms of the set of transport coefficients D, defined in the last line of eq. (1). We see that they are independent of the impurity content, up to an overall density prefactor coming from τ_{iz}. Note the appearance of the Pfirsch-Schlüter coefficient, $D_{PS} = (m_iT/e^2\tau_{iz})(\langle u^2B^2\rangle - \langle uB^2\rangle^2/\langle B^2\rangle) \geq 0$, which is seen to be sign definite by the Schwartz inequality. Thus $D_{11}^{zz} = -n_iD_{PS}/Z^2n_z$, and the impurity density gradient drives an impurity flux in the opposite direction, as required by entropy considerations. When the bulk ions are in the $1/\nu$ regime, $s = 0$ and $D_{11}^{zi} = -ZD_{11}^{zz}$. The flux driven directly by the electric field thus *vanishes*. We also see that $D_{12}^{z} = -(3/2)D_{11}^{zi}$, so there can be temperature screening, giving an outward impurity flux when $\eta_i \equiv \partial\ln T/\partial\ln n_i \geq 2$. In the lower collisionality $\sqrt{\nu}$ regime, the exact cancellation of the electric field drive coefficients is broken, leaving a drive dependent on the geometry function s, $\langle usB^2\rangle - \langle sB^2\rangle\langle uB^2\rangle/\langle B^2\rangle$. This is not sign definite and must be evaluated numerically for a given equilibrium, but we may expect it to be small in a well-optimised device. The relation $D_{12}^{z} = -(3/2)D_{11}^{zi}$ remains valid and so, depending on the sign of the geometric factor, either temperature screening persists, or the typically inward bulk ion density gradient will drive an additional outward impurity flux.

A recently developed numerical code, SFINCS, solves the drift kinetic equation with the full linearised Landau collision operator, in general stellarator geometry, for an arbitrary number of species [4]. The collisionality dependence of the transport coefficients of C^{6+} and Fe^{16+},

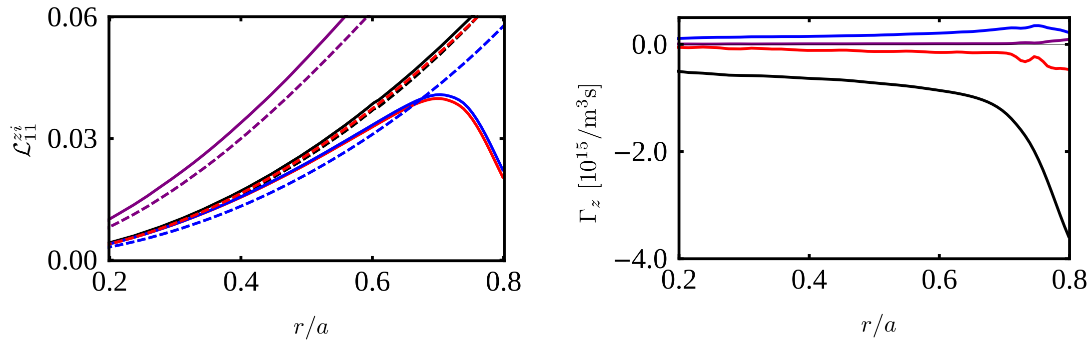

Figure 1: *Left: predicted* $\mathscr{L}_{11}^{zi} = (n_z Z e^2 \tau_{iz}/m_i p_i) D_{11}^{zi}$ *(black)* $E_r = 0$ *(dashed) and finite* E_r *(solid) compared to normalised SFINCS coefficients for* Fe^{24+} *in a* H^+ *plasma with* $Z_{eff} = 1.07$, $\mathscr{L}_{11}^{zi}$ *(red),* $-2\mathscr{L}_{12}^{zi}/3$ *(blue),* $-Z_z\mathscr{L}_{11}^{zz}$ *(purple). Right: Contribution to the* Fe^{24+}*flux from* dn_z/dr *(purple),* dn_i/dr *(red),* dT_i/dr *(blue),* Φ' *(black). Profiles against square root of the normalised toroidal flux.*

obtained with SFINCS for the standard configuration of W7-X, was presented in [1], supporting the analysis above. Here we have evaluated the radial profile of the transport coefficients of Fe^{24+} in a low collisionality H plasma, with plasma profiles characteristic of those expected in future high ion temperature discharges of W7-X [5]. The self-consistent electric field determined by a transport analysis is in the ion root, that is, pointing inwards across the discharge. The results are shown in Fig. 1, and compared to the values predicted by eq. (1). Setting the electric field to zero we see good agreement between the predicted and calculated profiles (the flux driven by finite A_{2z} is small). The optimisation of W7-X means that s is small, and the resulting small change in the predicted profiles is borne out by the simulation. Note that turbulent transport plays a role in setting the edge electric field, which may account for the distortion appearing here near the edge.

Thus impurity accumulation may still be averted in stellarators, but the net predicted flux depends sensitively on the plasma parameters and must finally be determined numerically. Note that the analysis presented is also relevant to the study of impurity behaviour in a tokamak, specifically in the core region where flux surface distortions commonly appear [6].

This work was supported by the Framework grant for Strategic Energy Research (Dnr. 2014-5392) from Vetenskapsrådet and part-funded by the RCUK Energy Programme [grant number EP/P012450/1]

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