Quasioptical approach to reconstruction of plasma fluctuations parameters using amplitude distribution of transmitted beam


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Introduction

Despite considerable progress in the theoretical understanding of confined plasmas, there is still a need for a detailed description of the fundamental physical parameters playing a role in the processes governing equilibrium, stability, and transport. Turbulence properties are some of the most important of such parameters.

Two basic approaches are used for remote sensing of the density fluctuation by electromagnetic beams. The first one is based on microwave reflectometry which basically allows the reconstruction of the turbulence spectrum from the spectrum of the scattered signal in the far zone [1]. The second one is phase contrast imaging (PCI), basing on reconstruction of the turbulence distribution from phase distortion of passing laser beams in geometrical optics zone [2]. In this work we present a theoretical approach to the reconstruction of inhomogeneous plasma properties from amplitude distribution of measured after pass through the plasma microwave beam in intermediate i.e. Fresnel zone. There are two promising advantages of this approach. First, the measurement of amplitude distribution in the beam transmitted through the plasma can be done easier than phase measurements required for PCI, because it does not require a matrix of separate receivers for each transverse point hence potentially has better spatial resolution. Second, amplitude perturbations in the Fresnel zone are of the first order of phase velocity perturbations, and the technique might allow detection fluctuations with low amplitude leading to weak and/or low angle scattering.

The discussed method is based on the quasi-optical approach to modeling the propagation of wave beams developed at IAP RAS, which allows taking into account the simultaneous effects of inhomogeneity and spatial dispersion of the medium, diffraction, aberration and dissipation of the wave beam. Based on this approach, a numerical code LAQO was developed. The code is able to model the propagation and the electron cyclotron absorption of wave beams in weakly relativistic plasma in electromagnetically large devices (with typical size of few thousands of wavelengths) in a reasonable time. Calculations using LAQO proved the importance of taking into account these effects to
describe correctly the RF power deposition profile in tokamak plasmas [3,4]. A small time of solving the direct problem of wave beam propagation allows the use of LAQO to solve the inverse problem, such as reconstruction of distributions of fluctuating plasma parameters using the amplitude profile of the transmitted beam. For solving the inverse problem we adopt the iterative method for the synthesis of the surface of the passive elements of microwave transmission systems developed in IAP RAS, which allows to achieve extremely high computational precision using a several iterations of direct problem calculations [5]. Note that this technique requires knowledge of phase distribution for the incident wave beam only, and for the transmitted wave beam only intensity distribution is needed.

**Reconstruction algorithm**

As the first step we present the results of reconstruction of the parameters of single perturbation of refraction coefficient in homogeneous isotropic media

\[
\delta n = \delta_0 \exp\left(\frac{-(\vec{r} - \vec{r}_0)^2}{a^2}\right).
\]

This simplest type of perturbation is characterized by amplitude \(\delta_0\), position \(\vec{r}_0\) and width \(a\).

The reconstruction routine uses the following algorithm. Given the amplitude and phase of the wave beam \((A_m, \phi_m)\) specified in start plane, a “measureable” amplitude distribution \(A_{\text{out}}\) in measurement plane is calculated for a wave beam transmitted through the media with refraction coefficient perturbation. Then the same calculation is done for the unperturbed media, and output amplitude and phase distributions \((A'_{\text{out}}, \phi'_{\text{out}})\) are obtained. After that, a substitution \(A'_{\text{out}} \rightarrow A_{\text{out}}\) is done, and the beam propagation is calculated in reverse direction. The amplitude distribution obtained in the latter calculation \(A'_{\text{in}}\) is substituted with the one taken from the incident beam, and the difference of phase distributions \(\phi'_{\text{in}} - \phi_{\text{in}}\) is stored as the phase corrector. This sequence is repeated until the amplitude distribution calculated with phase corrector and “measured” amplitude distribution converge with enough precision. Resulting phase corrector is converted into refraction coefficient perturbation. Note that distributions along and transversely to the beam are reconstructed essentially differently. Due to the fact that the phase corrector is 2-dimensional phase perturbation, the distribution along the beam is reconstructed
ambiguously. Therefore we have to use an additional scan over the position of the phase corrector for reconstruction of the longitudinal distribution of fluctuation.

**Reconstruction of transversal distribution of refraction coefficient perturbation**

Reconstructed refraction coefficient with parameters \( \delta_0 = 0.05 \), \( a = 1 \text{cm} \) is presented in figure 1. The wavelength was taken \( \lambda = 0.5 \text{cm} \), quasi-optical beam size at input \( a_{\text{beam}} = 3 \text{cm} \), the length of propagation track is \( l = 30 \text{cm} \). Refraction coefficient perturbation is located on the beam axis.

![Fig 1. Reconstruction of undisplaced perturbation. Right panel – distribution of refraction coefficient perturbation; central panel – reconstructed distribution of refraction coefficient perturbation; left panel – difference between true and reconstructed distributions.](image1)

One can see that position, amplitude and shape of perturbation in transverse plane are reconstructed with high precision, and resolution is less than the beam aperture. To illustrate this point, the reconstruction of “two-hump” perturbation is presented in figure 2.

![Fig 2. Reconstruction of refraction coefficient perturbation with two maximums. Right panel – distribution of refraction coefficient perturbation; central panel – reconstructed distribution of refraction coefficient perturbation; left panel – difference between true and reconstructed distributions.](image2)

**Reconstruction of longitudinal position of perturbation**

Proposed reconstruction procedure result in 2-dimensional phase corrector, which can be converted into refraction coefficient perturbation distribution with longitudinal uncertainty. It is unavoidable limitation because the original inverse problem is formally
incorrect. Our calculations, however, give some hope that the longitudinal position can be reconstructed with precision about the wave beam waist. To illustrate this, we made calculations with the same parameters as presented in figure 1, but with a shifted position of the phase corrector along the beam path back and forth from the position of the perturbation. The results are presented in figure 3. One can see that “inaccuracy” of the phase corrector position leads to the emergence of a strong Fresnel lens in the reconstructed phase corrector. Therefore, the condition of minimization of this lens allows reconstruction of longitudinal position of perturbation.

![Fig 3. Reconstruction results with different positions of phase corrector along the beam path. Right panel – before right position; central panel – in right position; left panel – after right position.](image)

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**References**