Modelling of tokamak plasma with SOL: MHD stability with finite
current density at the separatrix

A.A.Martynov¹, S.Yu.Medvedev¹,², A.A.Ivanov¹, Yu.Yu.Poshekhonov¹

¹Keldysh Institute of Applied Mathematics, Russian Academy of Sciences, Moscow, Russia
²National Research Nuclear University MEPhI, Moscow, Russia

For integrated modeling of equilibrium, stability and dynamics of the tokamak plasma with scrape-off layer (SOL) high resolution equilibrium calculations are needed. New version of the CAXE equilibrium code [1] computes the tokamak equilibrium on numerical grid adaptive to magnetic surfaces both in plasma with closed flux surfaces and in SOL region with open flux surfaces. The plasma profiles can be prescribed independently in each region with nested surfaces, and realistic SOL profiles with very small pressure drop off length can be accurately treated. From the point of view of the MHD equilibrium and stability modeling, self-consistent calculations of diverted tokamak configurations with finite current density at the separatrix require taking into account plasma outside the separatrix. The high resolution equilibria with SOL provide an input to new versions of the ideal MHD stability codes KINX and MHD_NX treating tokamak plasma with finite current density in SOL. Another possible application of the high resolution pedestal and SOL equilibrium code is a coupling to the SOLPS code with a purpose to increase equilibrium accuracy and support self-consistent transport/equilibrium modelling.

1 High resolution equilibrium with SOL currents

A starting point for high resolution fixed boundary equilibrium with plasma in SOL is a free boundary equilibrium with poloidal flux function $\psi$ available outside the separatrix. The values of $\psi$ from the free boundary equilibrium provide Dirichlet conditions at the boundary of the SOL region, private flux region and at the divertor plates for the solution of the Grad-Shafranov equation. The position of the X-point and the separatrix are free to adjust. In the new versions of the CAXE code [2] plasma profiles can be independently prescribed in each sub-domain with nested flux surfaces (3 – in case of single null, 4 – for double null assuming up-down symmetry) formed by the separatrix branches. The iterative four-color SOR solver allows for easy and flexible subdomain connectivity implementation and provides a reasonable code performance with a possibility of parallelization. The same boundary conditions can be used in computations of a series of high resolution equilibria with varying profiles in the plasma and SOL. As usual for the fixed boundary equilibria it implicitly assumes that external magnetic field adjusts to keep the poloidal flux distribution at the region boundary fixed under profile variations. At the same time it is likely that the separatrix shape change due to finite current density is driven by local equilibrium properties [3].
For the ITER Scenario 2 15MA plasma adding high pressure gradient in the narrow SOL region corresponding to the separatrix value $p_{sep} = 6.5$ kPa (giving 3 times higher pressure gradient in SOL compared to the pedestal for the drop-off length of $\sim 1.5$ mm [4]) does not change the equilibrium much (Fig.1a). Despite high pressure gradient at the separatrix the X-point angle is close to 90 degree provided that averaged parallel current density is low outside the separatrix (prescribed $< jB > / < B \cdot \nabla \phi >$ set to zero). However if the parallel current density is finite at the separatrix and continuously extended outside exponentially decaying even with a very short fall-off length, the change in the separatrix shape is significant (Fig.1b). For this purpose the profiles of the original ITER equilibrium were cut-off at the 0.995 fraction of the normalized poloidal flux inside the separatrix (Fig.1c) and expanded to a new separatrix. The corresponding changes in the connection length are demonstrated in Fig.1d. Significant change in the separatrix angle in the latter case results in about 10% increase in the connection length (localized in the region with finite current density in SOL) compared to the equilibrium with zero current density in SOL. In case of low parallel current density at the separatrix the large pressure gradient in SOL leads to a weaker decrease of the connection length but extended to a wider region in SOL.

![Figure 1. ITER 15MA equilibria with SOL and connection length calculations. (a) $p'$ contour plot for the case P3J0 with $j_{||} = 0$ in SOL and $p_{sep}=6.5$kPa, black line - separatrix for vacuum SOL; (b) $p'$ contour plot the case P1J1 with plasma profiles taken up to $\psi_b=0.995$, continuous $j_{||}$ and $p$, $p_{sep}=2.2$kPa, black line: separatrix for vacuum SOL(P0J0); (c) plasma profiles in pedestal (a.u.); (d) connection length from the equatorial plane to divertor for different cases.](image)

2 Kink-ballooning stability limits with SOL Modified KINX code [5] is used for detailed analysis of pedestal stability limits against medium-n kink-ballooning modes. The study of the pedestal limits in divertor tokamak plasmas with SOL extends investigations of the profile influence on the pedestal height [6]. The original ITER equilibrium features quite large pressure gradient at the separatrix. According to [6] it results in lower critical pedestal heights as compared to the case with low pressure gradient at the separatrix. Both pressure gradient and parallel current density were re-scaled in the pedestal by a factor of 1.25 to get an equilibrium unstable to the modes with toroidal mode numbers $n > 6$ (Fig. 2a,b) approximately matching the pedestal limit in the low pressure gradient case. As the SOL width with conducting plasma increases, the kink modes get eventually stabilized at the critical value. In Fig. 2c the critical width of
the SOL is plotted versus toroidal mode numbers in case of currentless SOL or assuming finite pressure gradient there giving an increase of the pressure at the pedestal top by about 6%. This is compared to the critical SOL width for the case with 8% larger pressure at the separatrix due to re-scaling of pedestal profiles (the case marked by PJC1.35 in Fig. 2). Note that for \( n < 15 \) the critical widths are close to each other for both cases. The case with finite SOL pressure is noticeably more unstable for \( n > 20 \) which can be attributed to the second stability access taking place for the re-scaled pedestal profiles. The growth rates moderately increase due to SOL pressure and decay fast if \( p' = 0 \) in SOL (Fig. 2d).

Figure 2. (a) Contour plot of plasma displacement projection normal to magnetic surfaces for \( n = 20 \) mode, growth rate \( \gamma/\gamma_A = 0.02 \), SOL width at equatorial plane 1 cm; (b) harmonics of the plasma normal displacement inside separatrix; (c) critical conducting plasma width outside the separatrix for the cases with currentless SOL (PJC1.25), with high pressure gradient in SOL (PJC1.25+PPSOLx3) and with pedestal profile rescale (PJC1.35); (d) growth rates versus the conducting plasma width for \( n = 10 \) and \( n = 40 \) for the cases with and without pressure gradient in SOL.

3 Double null NTT equilibria and SOL stabilization of \( n=0 \) peeling mode Double null negative triangularity (NTT-DN) equilibria with \( n = 1 \) external kink/Mercier mode limited \( \beta_N > 3 \) with high \( p' \) at the edge have been recently proposed as a prospective concept of tokamak power exhaust mitigation [9]. However the axisymmetric \( n = 0 \) stability with finite current density at the separatrix turned out to be an issue for all tokamak DN configurations in general due to destabilization of the "\( n = 0 \) peeling mode" [2].

The reactor size NTT-DN equilibria were computed with the 4 subdomain version of the KINX code using initial SPIDER [10] free boundary equilibrium. Let us note that for DN equilibria the HFS and LFS parts of the SOL are decoupled and the plasma profiles can be prescribed independently there. Ideal MHD stability calculations with SOL plasma are also performed with the new version of the MHD_NX code [7] on unstructured grids featuring now hybrid quadrangle/triangle mesh capability. On open field lines the sheath-compatible MHD boundary conditions [8] are used for the stability problem. The stability calculations with the MHD_NX code showed that the stability of the \( n = 0 \) peeling mode localized near separatrix (Fig. 3a) is not very sensitive to the presence of currentless plasma mantle separately either in the HFS or in the LFS part of SOL (Fig. 3b,c) – the growth rate decreases just by a factor of 2 (constant plasma density is assumed). The flow patterns of the modes feature reverse flow...
outside the separatrix. No unstable mode has been found with a presence of conducting plasma in either private flux region or in the HFS and LFS SOL regions at the same time at least in up-down symmetric configurations. Continuous pressure gradient across the separatrix just slightly changes the $n = 0$ mode growth rates. Finally the $n = 1$ kink/Mercier mode is not sensitive to the presence of conducting plasma outside the separatrix with growth rate decreasing just by 30% with conducting currentless plasma everywhere in SOL region (Fig. 3d).

![Diagram](image.png)

**Figure 3.** (a) Contour plot of toroidal electric field (proportional to the normal displacement in plasma) and streamlines of $n = 0$ peeling mode with vacuum outside the separatrix for the NTT-DN configuration, $\gamma/\gamma_A = 0.040$; (b) the same layout for the case with currentless plasma in HFS SOL with zoom to the X-point region, $\gamma/\gamma_A = 0.021$; (c) the same layout for the case with currentless plasma in LFS SOL with zoom to the X-point region, $\gamma/\gamma_A = 0.018$; (d) arrow plot of $n = 1$ kink/Mercier mode with currentless plasma everywhere outside the separatrix, $\gamma/\gamma_A = 0.017$.

4 Discussion and future work

New versions of the CAXE-SOL code compute high resolution equilibria under arbitrary pedestal and SOL profile variations. The method based on computational mesh adapted to magnetic surfaces provides fast and accurate solution, in particular suitable for stability calculations with finite current density in SOL and SOL transport studies coupled to quasi-equilibrium evolution. Kink-ballooning mode pedestal height limit is less sensitive to the pressure gradient profile in pedestal and SOL for medium-$n$ modes $n < 15$, i.e. pedestal pressure and not local gradient determines the height. Axisymmetric $n = 0$ X-point localized peeling mode for DN equilibria with finite current density at the separatrix can be stabilized by plasma outside the separatrix but specific conditions of stabilization need to be clarified. Another possible application of the developed codes is coupling to SOLPS [11].


Acknowledgement The work was supported by the grant from the Russian Science Foundation #16-11-10278.