Dynamic equilibria and MHD instabilities in toroidal plasmas with non-uniform transport coefficients

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Introduction

Plasmas confined by toroidal magnetic and electric fields are generally considered the best candidates to achieve sustainable nuclear fusion. Tokamaks and Reversed Field Pinches (RFPs) are two of such configurations which are currently investigated intensively. In the ideal case the plasmas in such reactors would remain quietly confined within the magnetic field in order to allow their core to reach the temperature needed for thermonuclear fusion.

To study fusion plasmas, magnetohydrodynamics (MHD) has been widely applied as an accessible description considering a force-free static equilibrium. Such an equilibrium only exists if the pressure forces are balanced by the Lorentz-forces that originate from the imposed magnetic and electric fields. In straight cylinder configurations such force-free states can be easily defined, considering for instance $z$-pinch and $\theta$-pinch devices \cite{1}. Even though such geometries can be subject to MHD instabilities, a force-free state can be defined. This changes in toroidal geometry.

In the simplest case in which the toroidal electric field is generated by a central solenoid, without external current drive, and where the toroidal magnetic field is induced by the poloidally orientated coils, ignoring ripples and other details, the imposed electro-magnetic fields have a simple form. It was shown in previous studies \cite{2, 3, 4, 5} that for this case, assuming uniform electric resistivity, such an equilibrium is not possible. These studies showed, following an increasing level of complexity, that the velocity can never be zero if the current density is linked to the electric field by Ohm’s law. The present work builds upon the results of these studies, increasing by one step the complexity, considering spatially non-uniform electric resistivity and viscosity profiles. Indeed, in practice, strong pressure, density and temperature gradients will influence the local values of the viscosity and resistivity in the plasma. The present approach takes this into account in the coarsest way, by defining profiles as a function of the minor radius. In particular we will show how different types of dynamic equilibrium and MHD instabilities...
appear as a function of the strength and the spatial distribution of the transport coefficients.

**MHD equations**

In the magnetohydrodynamics approximation, plasma is modeled as a charge-neutral electromagnetic conducting fluid. In the incompressible description, the dynamics are governed by

\[
\rho \frac{D\mathbf{u}}{Dt} = -\nabla p + \nabla \cdot \sigma + \mathbf{j} \times \mathbf{B}, \quad \frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E},
\]

(1)

\[
\mathbf{E} = \eta \mathbf{j} - \mathbf{u} \times \mathbf{B}, \quad \nabla \cdot \mathbf{u} = 0, \quad \nabla \cdot \mathbf{B} = 0,
\]

(2)

with the current density \( \mathbf{j} = \nabla \times \mathbf{B} \) and the pressure \( p \). These equations consist of dimensionless values, using the toroidal Alfvén speed \( C_A = B_0\sqrt{\rho \mu_0} \) as a reference velocity, with \( \rho \) the mass density and \( \mu_0 \) the magnetic permeability constant. The stress tensor \( \sigma_{ij} \) is given by

\[
\sigma_{ij} = \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right).
\]

(3)

In the case of a spatially uniform viscosity and density and resistivity, equation (1) is simplified to

\[
\frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho} \nabla p + \nu \Delta \mathbf{u} + \mathbf{j} \times \mathbf{B}, \quad \frac{DB}{Dt} = \mathbf{B} \cdot \nabla \mathbf{u} + \eta \Delta \mathbf{B}.
\]

(4)

The viscosity and resistivity profiles we consider in the present work mimic the profiles induced by the strong drop of temperature near the edge of experimental fusion reactors and are given by \( \eta(r) = \eta_0 f(r) \) and \( \nu(r) = \nu_0 f(r) \) with \( f(r) = \left(1 + (r/a)^8\right)^{5/4} \) where \( r/a \) is the normalized minor radius. In the following both uniform and non-uniform transport coefficients will be considered:

I. Constant magnetic-resistivity \( \eta_0 \) and kinetic-viscosity \( \nu_0 \),

II. Space dependent magnetic-resistivity \( \eta(r) \) and kinetic-viscosity \( \nu(r) \).

The profile of the magnetic-resistivity induces the profile of the current density through Ohm’s law (Eq.3) if \( \mathbf{u} = 0 \). We investigate the influence of the profile of the magnetic resistivity \( \eta \) on the dynamics, for a given fixed toroidal current, i.e. \( \int j_\eta(r) dV = \int j_\eta_0 dV \).

**Simulation results**

The calculations are performed for several toroidal magnetic field values \( B_{tor} \), ranging from 0.6 to 0.05 for Case I (uniform profile of coefficients) while keeping the poloidal current constant. The other, Case II, the toroidal magnetic field is ranging from 1.2 to 0.1. If the toroidal magnetic field \( B_{tor} \) is large enough, corresponding to the tokamak regime (\( B_{tor} = 0.6 \) for Case I and \( B_{tor} = 1.2 \) for Case II), the evolution of the magnetic field fluctuations is small and the
Figure 1: Isosurfaces for a value of 20% of the maximum toroidal velocity for case I,R (left) and Case I,T (right) (red contours show positive velocity and blue contours negative) at steady state for Lundquist number $M = 2000$. Also, a cross-section at a fixed value of the vertical coordinate is shown, quantifying the toroidal velocity in the plane [6].

Safety factor $q$ does not change much either. Concerning the shape of the $q$-profile in non-uniform resistivity case (Case II), it is more parabolic than in the uniform resistivity case (Case I), which is more realistic behavior for fusion plasmas [6]. By decreasing the toroidal magnetic field $B_{tor}$, to $B_{tor} = 0.05$ for Case I and to $B_{tor} = 0.1$ for Case II, MHD instabilities generate an important poloidal flow, and the dynamo process increases the toroidal magnetic field $B_{tor}$ at the core of the plasma. As a consequence, the plasma at the steady state shows the dynamics characteristic of RFP devices as shown in Fig. 1. Here, $M$ is the Lundquist number which is the dimensionless ratio of an Alfven wave crossing timescale to a resistive diffusion timescale. For both cases, the highest pinch-ratio case, where $B_{tor} = 0.05$ (Case I), $B_{tor} = 0.1$ (Case II), will be called the R-Regime (for RFP) and the opposite, $B_{tor} = 0.6$ (Case I), $B_{tor} = 1.2$ (Case II) the T-Regime (for Tokamak).

Figure 2 shows the visualizations of the velocity magnitude of Case I and II in the RFP-regime. The velocity gradients are more important in Case II. This is related to the fact that the comparison of the two cases is done for a given equal current. In the case of a non-uniform resistivity profile, the current is more concentrated in the center of the domain, so that the local pinch ratio will be higher. This generates a wider variety of unstable modes, as is reflected in the visualizations in Fig. 2.

Conclusions

The work presents a comprehensive numerical study of the dynamics of incompressible, visco-resistive MHD in a toroidal geometry. Both uniform and spatially variable viscosity and
Figure 2: Visualizations of velocity fluctuations at the steady state of the Lundquist number 2000 \( (M=2000) \) simulations of the RFP-regime, \( B_{tor} = 0.05 \) for Case I (left), and \( B_{tor} = 0.1 \) for Case II (right) [6].

... resistivity are considered. The resulting dynamics included dynamic equilibria for the quiescent, tokamak like regime, and the chaotic RFP-like regime.

In the present work it is shown how a spatial inhomogeneity of the viscosity and resistivity coefficients influences a toroidal equilibrium. Both parameters in a stable, tokamak-like regime and an unstable, RFP-like regime are considered. For a given total current, the velocities and magnetic fluctuations are more important by over an order of magnitude for the case where the transport coefficients are varying as a function of radius. Furthermore, the reversal parameter drops more importantly as a function of the pinch-ratio for the case of space-dependent coefficients. We have studied the parametric dependence of the Lundquist number \( (M) \) on the toroidal velocity and the poloidal velocity in detail in detail in [6].

References


