Control-oriented dynamical model of disruption generated RE beam

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We propose a simplified dynamical model of the RE beam current during the RE plateau such as

\[
\dot{I}_p(t) = -p_1 I_p(t) - p_2 \dot{I}_V(t) - p_3 \dot{I}_T(t) - p_4 FC(t),
\]

where the non-negative parameters $p_i, i = 1, \ldots, 4$ are identified via optimized algorithm based on prediction error minimization, $I_V$ and $I_T$ are the current flowing in the coils V and T shown in Figure 1, respectively, that are both magnetically coupled with the plasma/RE beam. The (real-time) Fission Chamber (FC) signal provides the counts of photoneutrons and photofissions induced by gamma rays with energy higher than 6 MeV (produced by bremsstrahlung of the RE interacting with the metallic plasma facing components). Identification results for a set of six disruption generated RE beams are shown in Figure 2. The parameters have been optimized for each single shot and their mean value is $[p_1, p_2, p_3, p_4] = [6.7, 7.5, 2.7, 1.3E − 8]$ with standard deviation $[1.6, 8.9, 1.8, 1.3E − 9]$. The standard deviation of the parameter $p_2$ is very large given that in some experiments where $I_V$ is constant $p_2$ is set to zero. The RE current decay time constant $1/p_1$ clearly depends on cold back-ground plasma density, beam anisotropy and inductance, high-Z impurity particles etc. as in [2, 3]. However we have decided in this preliminary study to neglect these dependences. This is consistent with the aim of designing a
robust controller able to cope with parameter uncertainties. Model refinements are the subject of ongoing studies. A standard PI controller plus a pre-programmed term, which resembles a feed-forward action, constitutes the plasma current control loop at FTU. It can be analytically proved that the single integral action of the PI controller is not sufficient to yield zero steady state error for constant desired plasma current. The error can even grow linearly when a ramp-up or ramp-down is required. Since our aim is the RE beam energy reduction by ramping-down its current, the latter issue strongly comes into play. Then, based on the proposed RE beam current approximate model, we have enhanced the PI controller adding a further block that implements a double integrator as shown in the right plot of Figure 1. The double integrator allows to im-

![Figure 2](image)

**Figure 2:** (Left: 6 plots) Comparison between experimental (solid) and estimated (dashed) RE beam current profiles. (Right) In the shot #38864 the double integrator is activated at 0.85s.

prove performances and the ramp-down tracking error amplitude is (theoretically) proportional to \(1/(p_3 \gamma_2)\). The value of the gain \(\gamma_2\) has been selected based on the identified range of the parameter \(p_3\). In [4] it has been shown how the external radius \(R_{ext}\) of the RE beam is a key feature at FTU to avoid RE interaction with the poloidal limiter during RE beam current ramp-down. In order to design a robust controller with improved \(R_{ext}\) tracking performances, we propose the following dynamical model in the **RE beam plateau phase** such as

\[
\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= -c_1 x_1 - c_2 x_2 + (c_3 (3.8 I_{V} + I_{F}) - c_4 I_p) + c_0 + c_5 x_3, \\
\dot{x}_3 &= -c_6 x_3 + (V_{loop} - c_7),
\end{align*}
\]

(2)

where \(x_1\) is the estimated \(R_{ext}\) and the parameters \(c_i, i = 1, \ldots, 7\) have been obtained with the same algorithms adopted for (1). \(\dot{x}_1 = \dot{x}_2\) consists of a term \((c_3 (3.8 I_{V} + I_{F}) - c_4 I_p) + c_0\) that takes into account the vertical magnetic fields produced by the current in the coil \(F, V, \) and \(I_p\), whereas the term \(c_5 x_3\) introduces to the Shafranov-shift since \(x_3\) is a rough estimate of the RE energy. The identification results are shown in Figure 3 and parameters mean and standard deviation are \([c_1, c_2, c_3, c_4, c_5, c_6, c_7] = [4.3E3, -2.9E6, -0.81, 23, 6.7E5, 6.6E - 3, 3.6] \) and \([1.8E3, 8.6E5, 1.2, 21, 1.6E5, 0.3, 2]\), respectively. Model refinements are under investigation to
Figure 3: Comparison between experimental (solid) and estimated (dashed) RE beam radial within the time intervals highlighted by red squares. (Right) In the shot #38864 the double integrator is activated improving the tracking performances.

decrease the variation of the parameters among different shots of the model (2).

We have analyzed the differential equation proposed in [2]

\[
\dot{W} = P_{\text{gain}} - P_{\text{coll}} - P_{\text{sync}} + P_{\text{th}} = e \frac{V_{\text{loop}}}{2\pi R} \cos(\theta) - \frac{n_e e^4 \ln(\Delta)}{4\pi \epsilon_0 m_e v} - \frac{2}{3} \gamma^4 \frac{1}{R^2(t)} + P_{\text{th}}(v),
\]  

that can be considered to estimate the energy dynamics of an electron as discussed in [2] where we added a further stochastic term \( P_{\text{th}} \) that dominates the other terms when \( v \) is low (\( W < 1 \text{keV} \)), i.e. when the electron is (thermal) and not runaway. This stochastic term has been designed [6] such that the density distribution of the random variable \( W \) satisfies the Maxwell-Boltzmann electron energy distribution. In (3), \( v \) is the electron velocity, \( \theta \) its pitch angle, \( n_e \) the electron density, \( \gamma = 1/\sqrt{1-(v/c)^2} \). RE in FTU are mainly generated by Dreicer effect, so no avalanche effects are taken into account. It is crucial to note that the number of roots of the right-hand side terms in (3), namely equilibrium points of the model, varies depending on the value of \( n_e \) and/or \( V_{\text{loop}} \). There are values of \( n_e \) (high) and \( V_{\text{loop}} \) (low) such that there is only one zero corresponding to an asymptotically stable (a.s.) equilibrium (only thermal electrons at stationary conditions). In the other hand with low \( n_e \) and high \( V_{\text{loop}} \) there exists again only one asymptotically stable equilibrium point at high energy: at steady state all electrons became runaway energy limited by synchrotron radiation loss. There are conditions in between in which three equilibria exist: two of them are a.s. and one, in the middle, is unstable. Then, at steady state, there are two stable (in random sense) population of thermal and runaway electrons depending on the past history of the system (energy history of each electron): this type of differential (stochastic) equation yields an hysteretic behavior of the runaway population. Note that similar results hold also in case of secondary RE generation mechanism as discussed (in a different approach) in recent works [1] and [5]. We show in Figure 4, for the first time to our knowledge, an experimental evidence of the hysteresis in case of RE generation and suppres-
Figure 4: (Left and middle column, top) Density in blue and loop voltage in red (scaled and offset); (middle) Neutrons plus gamma rays counts in red (NE213) and neutrons in blue (BF3); (bottom) Hysteresis graph of the estimated number of RE. (Right column, top) Simulation signal of density (blue) and loop voltage (red); (middle) Energy of 100 $e^-$ obtained integrating numerically (3); (bottom) Number of RE versus energy and time.

Further studies are necessary to adapt (3) to FTU machine and match experimental results: Figure 4 reports a simulation to show qualitative results obtained by (3) to show the formation and dissipation due to density changes of thermal and RE populations. A vertical displacement RE beam model and studies on magnetic coupling of RE with cold background plasma (magnetically confined by RE) as source of energy loss are currently under investigation to further refine the theoretical thresholds given in [7].

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References