Simple model of the discharge for Hall thrusters

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For some specific types of Hall thrusters the comparison of dependencies of the current of accelerated ions on the discharge voltage, which have been calculated within the limits of simple theoretical model, with the experimental data is presented. It is shown, that the offered simple model gives results close to a reality both qualitatively and quantitatively.

In paper \cite{1} a simple model for the determination of interrelation between integral characteristics of Hall thrusters with an anode layer is offered. We understand accelerating voltage, strength of a magnetic field, working gas flow rate, current of accelerated ions and geometrical sizes of the engine as integral characteristics. This model includes the equation of balance of the forces acting on the Hall current area, and the relation between the current of accelerated ions and working gas flow rate.

If the axial component of the magnetic field is either absent or weakly varying in the Hall current region, then the force exerted by the magnetic field on the “Hall current loop” is proportional to the difference between the magnetic pressures on both sides of the “loop” and the cross-sectional area of the ion beam. On the other hand, this force is proportional to the product of the current of accelerated ions and their momentum. This allows one to write an equation describing the balance of forces acting on the Hall current region. In so doing, it is assumed that the “Hall current loop” is imbedded in the equipotential anode plasma, while almost the entire potential difference is concentrated between the anode and cathode plasmas, i.e., the electric forces can be ignored.

Assuming that all ions pass across the total potential difference $\varphi_0$ applied to the discharge and have no angular scatter in velocities, the balance of forces can be expressed in the form

$$\frac{l}{c} J_i H_0 = \frac{M}{e} J_i \sqrt{\frac{2e\varphi_0}{M}},$$
where $M$ is the atomic mass of the working gas, $e$ is the elementary charge, and $J_i$ is the ion beam current, $c$ is the speed of light in vacuum, $J_H = \int j_H(r, z) dS \approx \Delta R \int_0^L j_H(z) dz$ is the total Hall current, $\Delta R = R_2 - R_1$, $R_2$ is the outer radius of the region occupied by the Hall current, $R_1$ is its inner radius, $l = \pi (R_1 + R_2)$ is the length of the midchord of the cross section of the region occupied by the Hall current, $L$ is the length of the region occupied by the Hall current along the $z$ axis, $H_0$ is the average magnetic field in the Hall current region.

In addition to this balance, another condition should be satisfied: the degree of ionization of working gas should be high enough to ensure the required value of the current of accelerated ions and, consequently, the above balance of forces.

It is obvious that the current of accelerated ions is proportional to the working gas flow rate

$$J_i = P Q,$$

where $Q$ is the flux of working gas in amperes. Coefficient of proportionality $P_i$ has the meaning of ionization probability of a working gas atom upon passing through the entire Hall current region. The high degree of ionization of the working gas in Hall thrusters is achieved because ionization is predominantly produced by the electrons forming the Hall current. We will assume that ionization is produced only by these electrons. The question of the ionization probability of neutral particles in a region with a known dependence of their depletion frequency on the coordinate has been considered in [2]. Using a simple model of gas discharge in crossed electric and magnetic fields, it is possible to find the relationship between the ionization frequency of the working gas and the integral characteristics of the thruster. Thus, the ionization probability turns out to depend on the ratio of the magnetic pressures on both sides of the “Hall current loop”, the accelerating voltage, the sort of gas, and the magnetic field.

As a result, we have two relationships that interrelate the accelerating voltage, the square root of the ratio of the magnetic pressures on both sides of the “Hall current loop” (the demagnetization parameter), the magnetic field strength, the current of accelerated ions for a given sort of working gas and given geometrical sizes of the Hall thruster. Excluding from them, e.g., the demagnetization parameter, we obtain the required relationship between the integral characteristics of the Hall thruster. In particular, for a specific thruster, this allows one to plot the dependence of the current of accelerated ions on the accelerating voltage at fixed
values of the magnetic field strength and the working gas flow rate, as well as the dependence of the current of accelerated ions on the magnetic field strength at fixed values of the accelerating voltage and the working gas flow rate, and to consider the question of the domain of existence of the Hall thruster discharge in the \((H_0, \varphi_0)\) coordinates. In spite of the fact that the proposed simple model was initially intended for Hall thrusters with anode layer (TAL), its application to stationary plasma thrusters (SPT) yields acceptable results.

The dependences of the current of accelerated ions on discharge voltage at constant value of the magnetic field strength, fixed gas flow rate and temperature of working gas \((T = 300\,\text{K})\) for TAL (Fig. 1A), SPT (Fig. 1B, 1C, 1D) and for hybrid Hall thruster (Fig. 1E) received using this method are presented in Fig. 1. The horizontal dotted straight lines correspond to the working gas flow rate, circles note their working points in the assumption of total ionization depletion of working gas. The curves 1 correspond to the hydrodynamic model of neutral particles motion; curves 2 - to the kinetic model with equal absolute values of initial velocities of neutral particles; curves 3 - to the kinetic model with Maxwellian distribution of initial velocities of neutral particles; curves 4 are the result of recalculation of experimental data from [3 - 8]. Thus, simple model offered in work [1] gives results close to a reality both qualitatively and quantitatively.
A simple model for the determination of interrelation between integral characteristics of Hall thrusters is proposed. Using this model, we obtained the dependence of the current of accelerated ions on the discharge voltage for some specific types of Hall thrusters. The proposed model can be used for preliminary estimates of the parameters of newly designed Hall thrusters.

References: