Interaction of explosive multiple filaments in magnetised plasmas

S.A. Henneberg[1], S.C. Cowley[2,3], H.R. Wilson[1]

[1] *York Plasma Institute, Department of Physics, University of York, Heslington, York, YO10 5DD, UK*

[2] *Culham Centre for Fusion Energy, Abingdon, Oxfordshire. OX14 3DB, UK*

[3] *Department of Physics, Imperial College, Prince Consort Road, London SW7 2BZ, UK*

E-mail: Sophia.Henneberg@york.ac.uk

1. Introduction

Two main operational regimes exist in tokamak fusion plasmas: the low-confinement mode (L-Mode) and the high-confinement mode (H-Mode). The H-Mode has a higher confinement than the L-Mode through a steep pressure gradient region at the edge, which creates a pedestal (Fig. 1). Fusion devices will likely operate in H-Mode because of its enhanced confinement. However the H-mode is vulnerable to violent plasma eruptions, called Edge Localised Modes (ELMs) which can be triggered by the steep edge pressure gradient. ELMs are quasi-periodic, filamentary instabilities which grow very rapidly [1]. They release a large amount of energy and particles which must be controlled to avoid excessive erosion of components on future fusion devices, such as ITER. Therefore it is very important to understand ELMs. Here, in particular, we are interested in how erupting plasma filaments with different heights influence each other's behaviour as this assists in understanding a more realistic early evolution of ELMs.

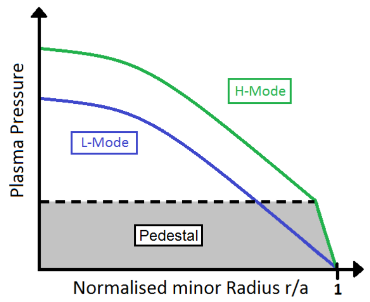


Figure 1: *Schematic pressure profile for L- and H-Mode*

2. Theoretical Model

Background

We employ a slab geometry which is susceptible to the Rayleigh-Taylor instability, [2, 3], see Fig. 2. The magnetic field lines $\mathbf{B_0}$ are in the z-direction and $|\mathbf{B_0}|$ varies in the x_0-direction. The density ρ_0 and the pressure gradient ∇p_0 are also only dependent on x_0. The gravity $\mathbf{g}$ points in the negative x_0-direction. The equilibrium is such that we have an unperturbed density and pressure at the z boundaries: $z = 0$ and $z = L$. We assume periodic boundary conditions in the y_0 direction, and we adopt a constant temperature along the field lines as the thermal conduction is fast along them. Here y_0

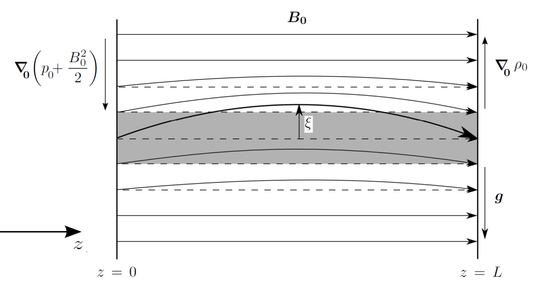


Figure 2: *Slab geometry: Straight field line represent the equilibrium.*

is equivalent to a poloidal angle in a tokamak plasma and x_0 is equivalent to the radial coordinate, ψ. The displacement perpendicular to the equilibrium magnetic field lines, ξ, of a field-aligned flux tube only depends on x_0, y_0 and t and is described by non-linear ballooning theory, [2]-[7]:

$$\overbrace{C_0\frac{\partial^2\xi}{\partial t^2}}^{\text{Inertia Term}} = \overbrace{C_1\left(1-\frac{(x_0-x_{max})^2}{\Delta^2}\right)\xi}^{\text{Linear Instability Drive}} - \overbrace{C_2\frac{\partial^2 u}{\partial x_0^2}}^{\text{Field Line Stability Term}} + \underbrace{C_3\xi\frac{\partial^2\overline{\xi^2}}{\partial x_0^2}}_{\text{Quasi-Linear-Nonlinearity Term}} + \underbrace{C_4\left(\xi^2-\overline{\xi^2}\right)}_{\text{Non-linear Growth Drive}} + \underbrace{\nu\frac{\partial^2}{\partial y_0^2}\frac{\partial\xi}{\partial t}}_{\text{Viscosity Term}} \tag{1}$$

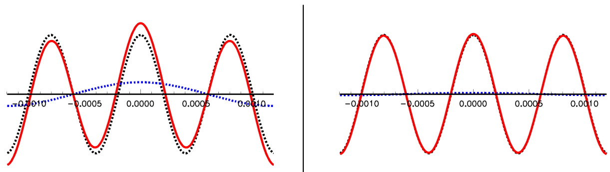
where $\xi = \frac{\partial^2 u}{\partial y_0^2}$ and an overbar indicates an average over the periodic spatial direction, y_0. The 0-subscript indicates that it is a Lagrangian coordinate, where $\mathbf{r} = \mathbf{r_0} + \xi$ and therefore the z-direction corresponds to an Eulerian coordinate [2, 3]. The impact of different equilibrium properties is captured in the coefficients C_0-C_4, the viscosity ν and the parameter Δ - see Ref. [5], [6] and [7] for details. For some types of tokamaks the order of the time derivative can also change, but for strongly shaped poloidal cross section this second order time derivative is appropriate [7]. Therefore this equation describing early non-linear evolution of ballooning modes has basically the same form for different geometries and therefore our results are generic.

Three Filament System

Solving the differential equation given by the linear terms of Eq. (1) which are highlighted in blue, we obtain the eigenmode solution which is used to initialise the simulation at sufficiently small amplitude h_i, so that the linear terms are initially dominant:

$$\xi_{init} = h_i\cos(n_i y_0)\exp\left(-\frac{(x_0-x_{max})^2}{2\sigma_i^2}+\gamma_i t\right) \tag{2}$$

with the toroidal mode number n_i, the linear growth rate $\gamma_i = -\frac{n_i^2\nu}{2C_0} + \sqrt{\frac{C_1}{C_0} - \frac{\sqrt{C_1|C_2|}}{C_0 n_i\Delta} + \frac{n_i^4\nu^2}{4C_0^2}}$ and the Gaussian width $\sigma_i = \sqrt{\frac{4\Delta}{n_i}\sqrt{\frac{|C_2|}{C_1}}}$. One can superimpose solutions with different mode numbers. Here we chose a two mode system with the following features: $n_2 = 3n_1$ and $h_2 = 50h_1$ which provides initial conditons with a main central filament which is only very slightly larger than the two side filaments, see Fig. 3a. In this case the difference in the filament amplitude is less than 2%, see Fig. 3b. A second simulation, which



(a) *Superpostition (red) of two modes with $n_2 = 3n_1$ and $h_2 = 5h_1$. Blue: n_1 and black n_2-mode. In the middle is the main central filament with two side filaments.*

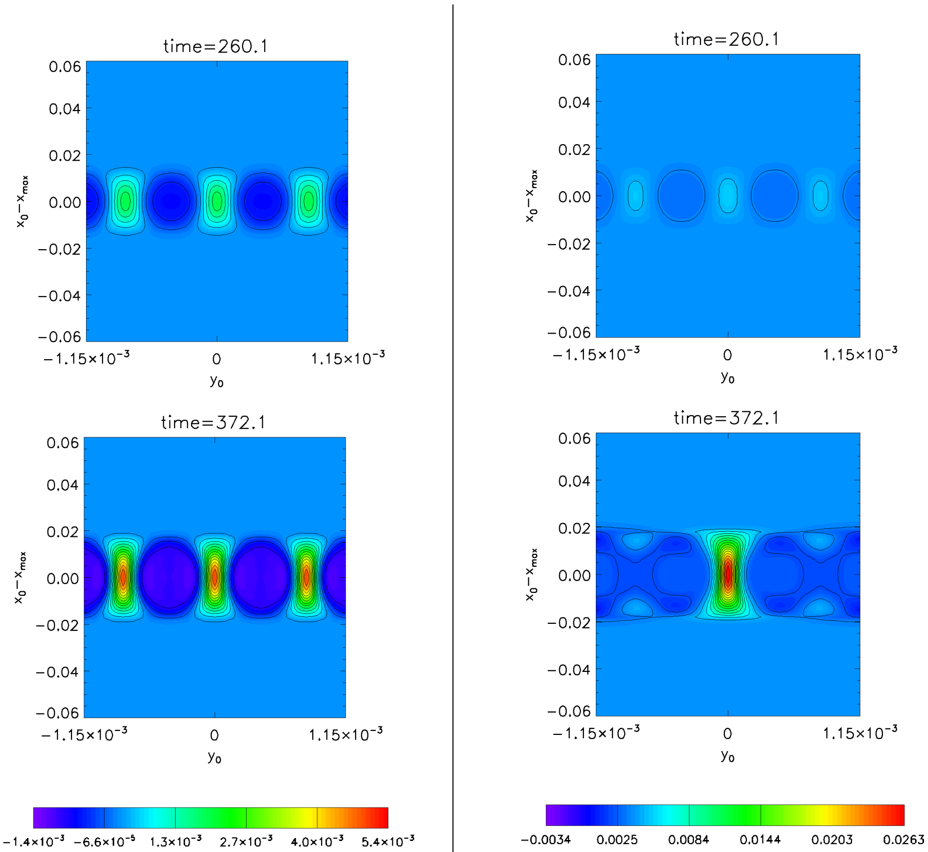
(b) *Superpostition (red) of two modes. This time with the actual heights $h_2 = 50h_1$ which produces a less than 2% larger main filament.*

Figure 3: *Initiation of the filaments for two different choices of relative amplitude.*

will be referred to as the single mode simulation rather than the two mode simulation, is performed with three equal sized filaments with mode number n_2.

3. Results and Discussion

Comparison between the two simulations described in the previous section shows that they evolve very similarly in the linear regime of the evolution, where the linear drive term of the ballooning mode envelope equation is dominant, see top Figs. 4a and 4b. However, after entering the non-linear regime,



(a) *Initialised with three equal sized filaments. The top is at the beginning of the non-linear regime. The bottom shows the end of the simulation)*

(b) *Central, main filament is initialised slightly larger (less than 2%) than the two side filaments. Same times as in 4a.*

Figure 4: *Evolution of the filaments.*

the side filaments are slowed down and then suppressed, see Fig. 5. Comparing the height of the main filament of the two-mode simulation with the heights of the filaments of the single mode simulation, one finds the main filament is larger by a factor of approximately 5 at the end of the simulation, see bottom Figs. 4a and 4b. Therefore one can conclude that the main filament is gaining height from the interaction with the side filaments, which are themselves suppressed. The non-linear terms of the ballooning mode envelope equation are responsible for the enhanced growth of the main central filament and the suppression of the smaller side filaments. The quadratic nonlinearity has an averaged ξ^2 which arises from a low order incompressibility condition of the plasma. For the largest filament $\left(\xi^2 - \overline{\xi^2}\right) > 0$ and acts as a drive, while for the side filaments $\left(\xi^2 - \overline{\xi^2}\right) < 0$ acts to damp the eruption. This term acts as a down draft created by the main filament eruption, which pushes back down on the side filaments. The quasi-linear (cubic) nonlinearity stabilises the filaments at the most unstable height, $x_0 = x_{max}$, but is responsible for the widening of the main filament radially and driving wings of the side filaments, see Fig. 4b.

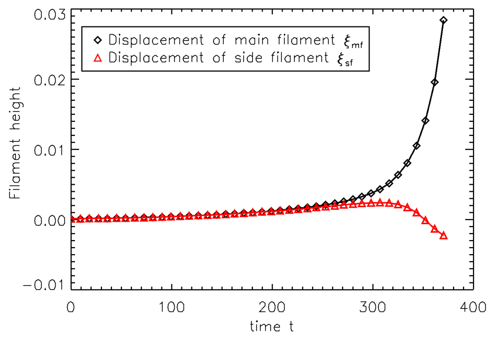


Figure 5: *Central filament (black) grows explosively. Side filaments (red) are suppressed in the non-linear regime.*

4. Conclusion and Future Work

We have shown how the interaction between filaments influences their evolution: Larger filaments gain amplitude from the interaction and grow faster than in the single mode case. Smaller filaments are suppressed in the non-linear regime. This interaction is caused by a down draft due to the incompressible plasma.

Future work is to calculate the coefficients C_0-C_4 and solve the non-linear ballooning equation, [7], for realistic tokamak geometry, including MAST and JET. This will allow a direct, quantitative comparison between theory and experimental observations.

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