Hydrodynamic analysis of self-similar radiative ablation flows

J.-M. Clarisse¹, J.-L. Pfister², S. Gauthier¹, C. Boudesocque-Dubois¹

¹ CEA, DAM, DIF, F-91297 Arpajon, France
² ENS Cachan, F-94235 Cachan, France

Hydrodynamic instabilities, and in particular ablation flow instabilities, are a key and with-
standing issue for laser-driven inertial confinement fusion (ICF). In an ongoing effort to obtain
a better description of the early-irradiation flow instabilities of an ICF pellet implosion (IPI),
a dedicated approach using, as mean flows, self-similar ablative heat-wave solutions of the gas
dynamics equations with nonlinear heat conduction [1] in slab symmetry, has been developed
and applied to direct laser irradiation configurations [2, 3]. This approach consists in comput-
ing, with a spectrally accurate numerical method [4], semi-infinite slab mean flows and their
time-dependent linear perturbations. Key features of real ablation flows such as unsteadiness,
confinement, compressibility that may be critical to the flow stability, are thus fully and accu-
ately taken into account. Under the assumption of radiative conduction and for positive pres-
sures and radiation fluxes at the slab external boundary, such self-similar solutions can be held
as being representative of the ablation of an ICF-pellet shell driven by a gas-filled hohlraum
radiation. A logical and preliminary step prior to any hydrodynamic stability study is then to
perform a hydrodynamic analysis of such mean flows. In doing so, a framework for analyzing
more general (i.e. not necessarily self-similar) unsteady ablative mean flows is provided.

Dimensionless equations of motion

The proposed model flow considers the motion of an inviscid, heat-conducting fluid with a
polytropic equation of state, \( p = R \rho T, \) \( \varepsilon = RT / (\gamma - 1) \), and a nonlinear heat conductivity of
the form \( \kappa = \kappa_0 (\rho_0 / \rho)^\mu (T / T_0)^\nu, \kappa_0 > 0, \mu \geq 0, \nu > 0, \) where \( \rho_0 \) and \( T_0 \) are homogenization
constants. The fluid which is initially at rest, of uniform finite density \( \rho_f \), and occupying the
half-space \( x \geq 0 \), is set into motion by time-increasing pressure and heat flux applied at its
external boundary surface \( \Gamma \). Relying on the physical governing parameters \( \gamma, R, \kappa_0, \rho_f, \) characteristic boundary pressure \( p_\Gamma \) and heat flux \( \varphi_t \), supplemented by a reference time \( t_c \), two
dimensionless formulations of the equations of motion in Lagrangian coordinates are proposed:

\( (\mathcal{B}_p, \mathcal{B}_\varphi)\)-formulation [2]

\[
\begin{align*}
\frac{\partial}{\partial t} \left[ \frac{1}{\bar{p}} \right] - \partial_t \bar{v}_x &= 0, \\
\frac{\partial}{\partial t} \left[ \frac{1}{2} \bar{v}_x^2 + \frac{1}{\gamma - 1} \bar{T} \right] + \partial_t [\bar{p} \bar{v}_x + \bar{\varphi}_x] &= 0, \\
\bar{\varphi}_x &= -\bar{p} \kappa \partial_m \bar{T},
\end{align*}
\]

\( (\mathcal{M}, \mathcal{Pe})\)-formulation

\[
\begin{align*}
\frac{\partial}{\partial t} \left[ \frac{1}{\bar{p}} \right] - \partial_t \bar{v}_x &= 0, \\
\frac{\partial}{\partial t} \left[ \frac{1}{2} \bar{v}_x^2 + \frac{1}{\gamma - 1} \bar{T} \right] + \frac{1}{\gamma - 1} \partial_m (\bar{p} \bar{v}_x + \bar{\varphi}_x) &= 0,
\end{align*}
\]
with $\bar{p} = \bar{\rho} \bar{T}$ and $\bar{k} = \bar{\rho}^{-\mu} \bar{T}^\nu$, and where $\bar{m}$ is such that $d\bar{m} = \bar{\rho} d\bar{\xi}$. Driving dimensionless parameters, apart from $\gamma$, are ascribed with the former formulation to the sole boundary condition parameters $B_p$ and $B_\phi$ (cf. Tab. 1), while they are, with the latter, encharged in the governing equations under the form of Mach and Péclet number scales $\mathcal{M}$ and $\mathcal{Pe}$. For initial and bound-

<table>
<thead>
<tr>
<th>$(B_p, B_\phi)$</th>
<th>$\bar{\rho}(\bar{m}, 0)$</th>
<th>$\bar{V}(\bar{m}, 0)$</th>
<th>$\bar{T}(\bar{m}, 0)$</th>
<th>$\bar{p}(0, \bar{t})$</th>
<th>$\phi(0, \bar{t})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\mathcal{M}, \mathcal{Pe})$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>$B_p \bar{T}^{2(\alpha-1)}$</td>
<td>$B_\phi \bar{T}^{3(\alpha-1)}$</td>
</tr>
</tbody>
</table>

Table 1: Invariant-density self-similar solutions. Initial and boundary conditions.

ary conditions compatible with a reduction of variables (Tab. 1), these formulations lead to two complete classifications of self-similar solutions (cf. Tab. 2).

<table>
<thead>
<tr>
<th>$B_p$</th>
<th>$B_\phi$</th>
<th>$\mathcal{M}^2$</th>
<th>$\mathcal{Pe}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\gamma - 1) \gamma^{-2} \mathcal{Pe} \mathcal{M}^{-2}$</td>
<td>$(\gamma - 1)^{3/2} \gamma^{-5/2} \mathcal{Pe}^{3/2} \mathcal{M}^{-2}$</td>
<td>$\gamma^{-1} B_\phi B_p^{-3}$</td>
<td>$\gamma(\gamma - 1)^{-1} B_\phi^2 B_p^{-2}$</td>
</tr>
</tbody>
</table>

Table 2: Relations between the driving parameters $(B_p, B_\phi)$ and $(\mathcal{M}, \mathcal{Pe})$.

**Invariant-density self-similar radiative ablation flows**

Self-similar solutions leaving invariant the flow density correspond to the similarity exponent definition $\alpha = (2\nu - 1)/2(\nu - 1)$ and the similarity laws of Tab. 3, the systems of PDEs being replaced by systems of ODEs for the flow reduced variables, with initial and boundary conditions of Tab. 1 being transformed, respectively, into boundary conditions at $\xi = +\infty$ and at $\xi = 0$ [2]. For sufficiently low values of the external boundary heat flux, the heated fluid

region may be considered to be bounded by a non-isothermal shock-wave discontinuity [1, § C] and the boundary conditions at $\xi = +\infty$ replaced by the relevant Rankine–Hugoniot jump relations at the shock-front location (e.g. [2]). Self-similar radiative ablation flows within this bounding shock-wave approximation have been investigated — for a monatomic gas ($\gamma = 5/3$) and the fully ionized gas model of Kramers ($\mu = 2$, $\nu = 13/2$) — by means of an extensive exploration of the space of parameters $(B_p, B_\phi)$ for solutions computable with the numerical method detailed in [4]. A set of 631 ablative solutions, presenting a large variety of spatial pro-
files (Figs. 1a, c, e, g) and including a solution extrapolated from an IPI simulation with an ICF hydrodynamics code (IPI-like flow), has thus been obtained.

Hydrodynamic analysis

The analysis of these ablative flows has been performed in the reference frame of the ablation front (af) — here ascribed to be the locus of the minimum temperature-gradient length, $\ell_T = \min_x \ell_T$ — leading to the definition of local Mach, Péclet, stratification and Froude numbers in terms of the fluid velocity relative to the ablation front $v'_s$, the ablation-front acceleration $a_{af}^s$, and proper local gradient lengths (Tab. 4). Completed by the flow-region relative lengths $\ell_{\text{cond}}/\ell_{\text{tot}}$ (conduction region), $\ell_T/\ell_{\text{tot}}$ (ablation-front stiffness), and $(1 - \ell_{\text{cond}}/\ell_{\text{tot}})$

<table>
<thead>
<tr>
<th></th>
<th>Mach</th>
<th>Péclet</th>
<th>stratification</th>
<th>Froude</th>
</tr>
</thead>
<tbody>
<tr>
<td>flow analysis</td>
<td>$M = \frac{</td>
<td>v'_s</td>
<td>}{\sqrt{T_0/\rho}}$</td>
<td>$Pe = \frac{</td>
</tr>
<tr>
<td>LM criteria</td>
<td>$\gamma M^2 \ll 1$</td>
<td>$\frac{T}{T-1} M^2 \ll 1$</td>
<td>$Sr' = \frac{\ell_T a_{af}^s}{p/\rho}$</td>
<td>$\frac{\gamma M^2}{Fr^2} (= Sr') \ll 1$</td>
</tr>
</tbody>
</table>

Table 4: Hydrodynamic characteristic number definitions and LM approximation criteria.
(compressed-fluid region), this analysis yields a global and detailed description of the flow hydrodynamic features. For example, strong compressibility effects — fast expansions of the heated fluid \( \ell_{\text{cond}}/\ell_{\text{tot}} \approx 1 \), Figs. 2a, b), isentropic Chapman–Jouguet (CJ) points \((M = 1, \text{Figs. 1b and 2c, d})\), ablation fronts with high stiffnesses and strong dominance of pressure gradients over inertial forces \((Sr_{af} \ll 1, \text{Fig. 1f})\) — are evidenced for sufficiently large values of the Péclet scale \( Pe = \propto B^2_{\varphi}/B^2_p \), and in particular for the IPI-like flow. Regarding the

![Graphs](image-url)

Figure 2: Radiative self-similar flow. Hydrodynamic characteristic numbers (IPI-like flow □).

low-Mach-number (LM) approximation commonly used in ablative Rayleigh–Taylor instability modeling, none of the flows satisfies the Froude criterion \( \gamma M^2/\text{Fr}^2 = S r' \ll 1 \) (see Tab. 4) in the vicinity of the external surface and of the ablation front (Fig. 1h), while flows with CJ points obviously do not fulfill the low-Mach criterion over part of their conduction region (Fig. 1b). For arbitrary time-dependent flows, the same set of characteristic numbers is equally relevant, the corresponding analysis having to be performed repeatedly over time.

References


