Estimation of Wendelstein 7-X magnetic field perturbation due to permeable materials
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Wendelstein 7-X (W7-X), currently under commissioning at the Max-Planck-Institut für Plasmaphysik in Greifswald, Germany, is a helical advanced stellarator, combining the modular coil concept with optimized plasma properties. To achieve the designed magnetic field structure and reliable operation of the machine not only a high accuracy of coil manufacturing and assembly is required, but also a careful assessment of the influence of all permeable materials in the vicinity of the plasma. This paper discusses the simplified modelling of the permeable material influence on the magnetic field perturbation which can be relevant also for other devices aiming to assess more accurate the magnetic field by consideration of the impact of permeable materials and presents a typical example of such calculations for Wendelstein 7-X.

Keywords: Wendelstein 7-X, stellarator, permeable materials, magnetic field perturbation

Introduction
During a construction of Wendelstein 7-X (Figure 1) a multitude of different materials is used, which should be compatible with the experimental conditions on one hand and ensure a proper operation on the other hand. To guarantee the unperturbed topology of the magnetic field the handling of permeable materials in the W7-X hall and boundary conditions for their application are strictly regulated by the corresponding technical guideline. It requires to use materials with magnetic permeability < 1.01 within the outer vessel area. Only for the welding seams this limit is relaxed up to 1.05. In all cases deviating from these specified requirements the impact on the magnetic field perturbation must be accurately estimated taking into account the exact location and the specific geometry of a permeable machine component or its pieces. This analysis is a basis for the further release procedure of a permeable material, registered afterwards in the corresponding data base, which allows one to follow the details of the application whenever necessary. The obligatory condition for the allowance to use a permeable material is that the
magnetic field perturbation $B_s/B_0$ should be less than $1 \cdot 10^{-4}$, where $B_s$ – is a perturbed magnetic field, $B_0$ – an unperturbed magnetic field at the location of interest/computation point (plasma edge).

**Modeling methods**

There are several possibilities to estimate the magnetic field perturbation $B_s/B_0$. The simplest method, applicable for a small permeable component located far away from the plasma, is to use the analytical approximation of one permeable sphere [1]:

$$
B_s/B_0 = 2 B_k/B_0 \left( \mu_r - 1 \right) / \left( \mu_r + 2 \right) \cdot (r_k/d)^3, \tag{1}
$$

where $B_k$ – magnetic field at the location of the permeable sphere, $\mu_r$ – relative permeability of the considered material, $r_k$ – radius of the permeable sphere with the volume equal to the volume of the considered permeable component, $d$ – distance between the permeable sphere and the location of interest. In this approximation the angle between the local magnetic field and the vector along the distance $d$ is neglected.

Another possibility is to use finite element (FE) calculations, for example with help of ANSYS code. This method should be used when considering the non-linear dependencies of the magnetic permeability and for complicated geometries of the permeable components especially when an accurate modelling of the magnetic field perturbation near the allowable limit is necessary. The disadvantage of this method is that FE calculations are very time consuming.

To find a compromise between both needs (to perform calculation in reasonable time and to avoid too rough approximation) the method of mutually interacted spheres was developed for W7-X tasks and realised in the program “W7-X permeabilities” (author:
The magnetization $M$ of a permeable sphere in an external homogeneous magnetic field $B_0$ is

$$M = \frac{3}{\mu_0} \left( \frac{\mu - \mu_0}{\mu + 2\mu_0} \right) B_0,$$

where $\mu$ - permeability of the sphere, $\mu_0$ – permeability of vacuum. The magnetic scalar potential is given by

$$\phi_M = \frac{1}{3} M a^3 \cos \phi \frac{1}{r^2},$$

where $a$ - radius of the sphere, $r$ – distance between the sphere and computation point, $\phi$ - angle between the magnetic field of the sphere and the distance vector between the sphere and the computation point. The vector of the magnetic field strength is given by $H = - \nabla \phi_M$. The magnetic field components are defined now from the vector equation: $\mathbf{B} = \mu_0 \mathbf{H}$.

The initial magnetic field $B_0$ can be calculated by Biot–Savart law. Then the perturbed magnetic field is calculated for every out of N mutually interacting spheres, which gives 3N equations written above. Afterwards the magnetic field perturbation at each computation point is found as a superposition of the local initial field $B_0$ and a sum of local perturbations from all spheres. This algorithm is realised in MATLAB package which allows a “matrix-division” (a possibility to derive vector $\mathbf{x}$ like $\mathbf{x} = \mathbf{A}^{-1} \mathbf{b}$ from the equation $\mathbf{A} \mathbf{x} = \mathbf{b}$, where $\mathbf{A}$ is a matrix and $\mathbf{b}$ is a vector).

The complex geometries can be modelled in this method with help of many spheres, where each sphere is small enough to consider the field $B_0$ as homogenous. The method of mutually interacting spheres is much more accurate than the approximation (1) and is extremely fast in comparison to FE calculations (100s against 2 days for a task with 2500 spheres). This allows checking the magnetic field perturbation for all reference W7-X configurations [3] and estimating its maximum value in a reasonable time. Result inaccuracies depend on how conservative is the geometry model.

**One example for Wendelstein 7-X applications**

For the user convenience the MATLAB program realizing the method described above includes a graphical user interface (GUI). The input data consist of the geometry description of permeable components approximated with spheres, the radii of these spheres and their magnetic permeability.

One typical task during Wendelstein 7-X construction was the estimation of the magnetic field perturbation caused by magnetic parts of cooling circuits with an increased permeability. Each circuit part was represented by several spheres distributed between its ends. The radii of the spheres were calculated from the corresponding volumes of the circuits. The
magnetic permeability value was chosen conservative, equal to 2000. The maximum magnetic field perturbation was chosen among calculations for 18 operational cases (9 reference operational cases x 2 directions of currents in trim coils). The resulting perturbation value of $5.45 \cdot 10^{-7}$ was significantly less than the specified allowable limit of $10^{-4}$.

Figure 2 represents the GUI interface during the calculation showing the modeled geometry and the magnet system of W7-X. Figure 3 shows the value of the maximum magnetic field perturbation and its distribution at the plasma edge.

**Summary**

To achieve the designed magnetic field structure and reliable operation of the Wendelstein 7-X careful assessments of the influence of all permeable materials in the vicinity of plasma are necessary. For this purpose the official procedure for handling the permeable materials was established based on the corresponding technical guideline. The method of mutually interacting spheres is a reasonable compromise between analytical approximations and FE calculations, and allows performing quick and reliable assessments. All results of calculations are stored in the W7-X documentation system. Material data are collected in the corresponding data base, giving an overview about main properties of any specific component, where the magnetic permeability does not comply with guideline requirements.

**References**