Convective and diffusive loss of fast ions in tokamaks

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1. Introduction

Here we investigate how the combined action of radial convection and diffusion of charged fusion products (CFPs) influences the CFP losses in tokamaks. Our treatment is based on a simplified 1D Fokker-Planck (FP) model for fast ion transport in quasi-steady-state tokamak plasmas. The paper examines the effects of convective and diffusive transport on the radial density profiles of CFPs at the plasma edge as well as on the energy distribution of lost CFPs at the first wall. The importance of considering simultaneous radial diffusion and convection is pointed out for the interpretation of fast ion loss measurements in tokamaks [1-3], especially for explaining the anomalous loss (additional to first orbit loss) of charged DD fusion products in JET [4,5].

2. Model Fokker-Planck equation

Referring to a tokamak with weak TF ripples, we base our study on the drift FP equation for the fast-ion distribution f in the 3D constants-of-motion (COM) space $\boldsymbol{c} = \{E,\lambda,r_m\}$ [3], i.e.

$$\partial_t f = \nabla_{\mathbf{c}} \cdot \left(\mathbf{d}_{\mathbf{c}} - \overleftrightarrow{\mathbf{D}}_{\mathbf{c}} \cdot \nabla_{\mathbf{c}} \right) f + S\left(\mathbf{c}\right) \ , \tag{1}$$

where E and λ are the particle energy and normalized magnetic moment, r_m is the maximum radial coordinate along the orbit, $\mathbf{d}_{\mathbf{x}}$ and $\overleftrightarrow{\mathbf{D}}_{\mathbf{x}}$ describe the convective and diffusive collisional transport of fast ions associated with slowing down and, respectively, pitch-angle scattering, and $S(\mathbf{c})=S_0(\mathbf{c})\delta(E-E_0)$ is the fast-ion source term with E_0 representing the birth energy.

2.1 Simplified FP description of slightly thermalized CFPs

For the nearly isotropic ($\partial_\lambda \cong 0$) slightly thermalized CFPs ($E \sim E_0$), confined in quasi-steady-state tokamak plasmas ($\partial_t \cong 0$), Eq. (1) can be reduced to

$$0 = \partial_E g^{1/2} d_E f + \partial_{r_m} g^{1/2} \left(d_{r_m} - D_{r_m r_m} \partial_{r_m} \right) f + g^{1/2} S\left(\mathbf{c}\right), \tag{2}$$

where $g^{1/2}$ is the Jacobian. Introducing an effective time variable τ=1-E/E_0 and the normalized radial coordinate x =1- r_m/a, where a stands for the first wall radius, we arrive at the following simplified 1D Fokker-Planck equation for energetic CFPs:

$$\partial_\tau f = -\partial_x \left(d + D\partial_x \right) f + f_0 \delta\left(\tau\right) \tag{3}$$

with $d = \left[d_{r_m} / \left(a d_E \right) \right]_{r_m = a, E = E_0}$ and $D = \left[D_{r_m} / \left(a^2 d_E \right) \right]_{r_m = a, E = E_0}$ representing the radial convection and diffusion coefficients, respectively, at the variables' boundaries, and $f_0 = \left(S_0 / d_E \right)_{E = E_0}$ being the distribution function at birth energy. In obtaining Eq. (3) we took also the Jacobian at $\tau = x = 0$. We note that the above assumptions are valid for the considered slightly-thermalized marginally confined fast ions, i.e. for particles from the narrow ranges $a\text{-}r_m << a$ and $E\text{-}E_0 << E_0$. Taking into account that $f \rightarrow 0$ at the plasma edge as well as of the disappearance of the radial flux in the plasma core ($df + D\partial_x f \rightarrow 0$) we apply the boundary conditions

$$f\left(x,\tau\right)\Big|_{x=0} = 0, \quad f\left(x,\tau\right)\Big|_{x\to\infty} = f_0\left(x=0\right)\eta\left(x\right), \tag{4}$$

where $\eta(x)$ is the Heaviside step function. For the purpose of a compact analytical solution of Eq. (3) we extended the radial coordinate to $x = \infty$, which does not affect the slightly-thermalized marginally confined CFPs. A solution of Eq. (3) under conditions (4) is found as

$$f = \exp\left(-y/2\right) \sum_{\sigma=\pm1} \Psi_\sigma\left(y,z\right), \quad \Psi_\sigma = \sigma \exp\left(\sigma y/2\right)\left\{1 + \operatorname{erf}\left[\left(z - \sigma y/z\right)/2\right]\right\}/2 \tag{5}$$

with $y = dx/D, \ \ z = d\sqrt{\tau/D}$, erf($x$)=error function.

2.2 Radial profiles of slightly thermalized CFPs

Fig. 1a demonstrates the radial distributions of slightly thermalized CFPs, Eq. (5), at different energies in the case of pure diffusive transport. The radial diffusion is seen to considerably influence the density profiles of confined CFPs at the plasma periphery. The effect of radial

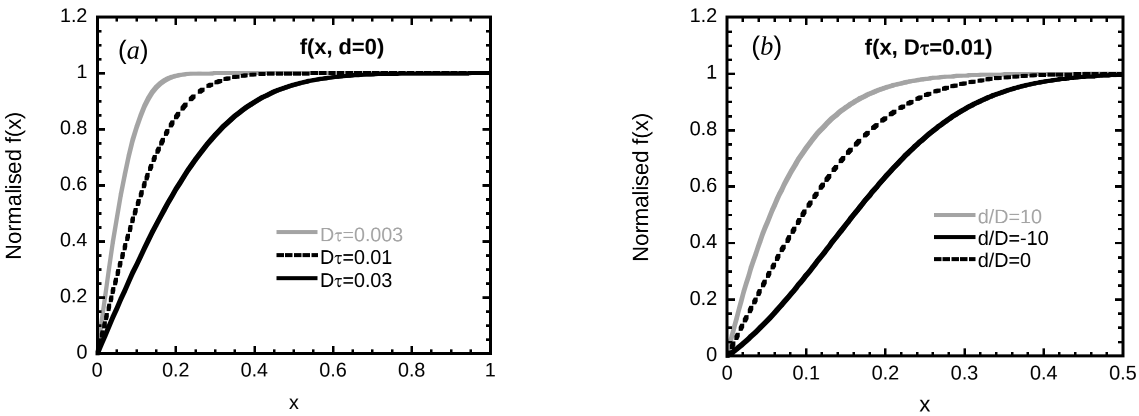

Fig. 1: (a) Radial profiles of the charged fusion product distribution at different values of $D\tau$ in the case of pure diffusive transport, (b) Density profiles of partly thermalized CFPs ($D\tau$=0.01) depending on the value of radial convection characterized by the ratio d/D.

convection at $D\tau = 0.01$ is displayed in Fig. 1b. In the case of outward convection ($d/D = -10$) a substantial degradation of the confinement of peripheral fast ions is evident, whereas for inward convection ($d/D = +10$) the fast ion confinement appears improved.

2.3 Energy spectra of lost slightly thermalized CFPs

Knowing $f(x,\tau)$ we can express the flux of CFPs lost through the considered radial transport as

$$\gamma(\tau) = D\partial_x f(x,\tau)\big|_{x=1} = 2d\left[\frac{\exp(-T^2)}{\sqrt{\pi}T} + \mathrm{erf}(T)\right] + 2d, \quad T = \frac{d}{2D}\sqrt{\tau} \equiv \frac{d}{2D}\sqrt{1-\frac{E}{E_0}}, \tag{6}$$

and, correspondingly, the total number of particles lost at energies above E as

$$l(E) \propto \int_0^{\tau(E)} d\tau \Gamma(\tau) \propto (1+2T^2)\mathrm{erf}(T) + 2T\left(T + \frac{\exp(-T^2)}{\sqrt{\pi}}\right). \tag{7}$$

These quantities $\gamma(E)$ and $l(E)$ represent the flux and the number of lost CFPs in the case of a monoenergetic source $S(E)=S_0\delta(E-E_0)$. A non-monoenergetic fusion source $S(E-E_0)$ may be accounted for by integrating $\hat{\gamma}(\tau)$ to obtain the loss flux

$$\Gamma(E) = \int_E^{\infty} dE' S(E'-E_0)\gamma(\tau'), \quad \tau' = \frac{E'-E}{E_0}. \tag{8}$$

Figure 2 displays the convective-diffusive flux of lost CFPs and the ratio of this flux to the pure diffusive loss flux for different typical ratios of convective and diffusive rates. A strong effect of radial convection is seen on both the number and the energy spectrum of lost fusion

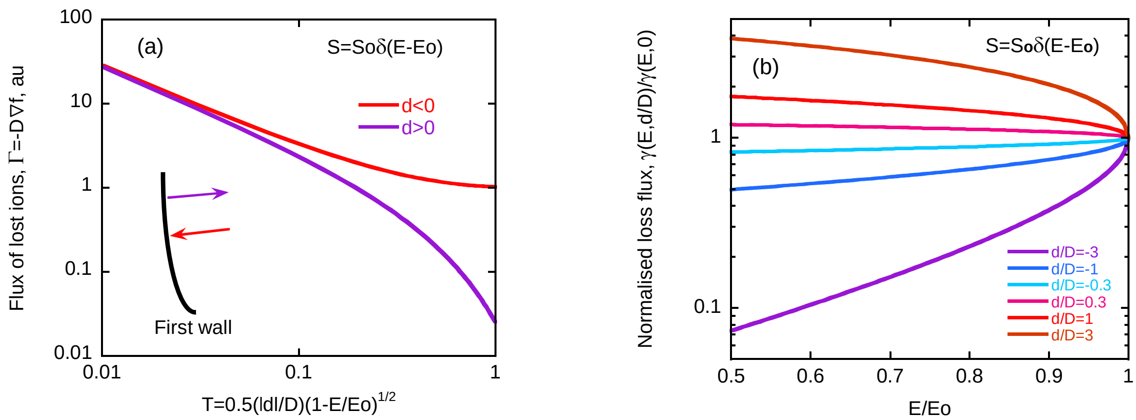

Fig. 2: (a) Fast ion loss flux depending on energy for inward and outward convection, (b) Normalized CFP loss flux for typical ratios of convection and diffusion rates.

products. The total number of monoenergetically produced CFPs which are lost at energies exceeding energy E, is shown in Fig. 3. There the outward convection is seen – in comparison to inward convection – to result in a ~ 6-fold enhancement of the number of lost particles when $1- E/E_0 \sim 4D^2/d^2$. To evaluate the impact of wide energy spectra on the CFP loss we consider, as a model non-monoenergetic source, the Gaussian shape

$$S(E) \propto \exp\left[-\frac{(E-E_0)^2}{(\Delta E)^2}\right], \quad \Delta E \cong \sqrt{3T_i E_0}, \tag{9}$$

where T_i is the effective temperature of the parent ions. The energy spectra of the pure diffusive flux of lost CFPs as well as the first orbit loss flux for T_i=0.05E_0 are shown in Fig. 4, for which we supposed the FO loss l_{FO}, i.e. collisionless loss of CFPs, to scale with energy according to a weighting factor E^2 multiplied to the source distribution.

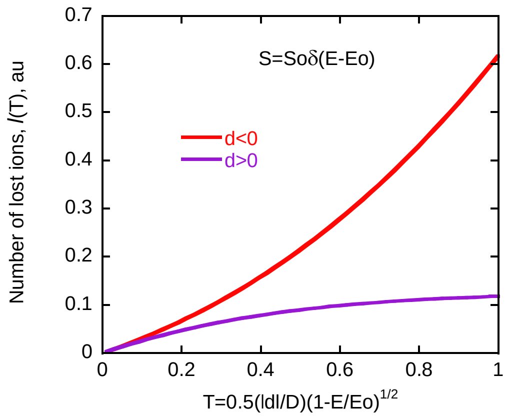

Fig. 3: Total number of CFPs lost at energies exceeding a certain E.

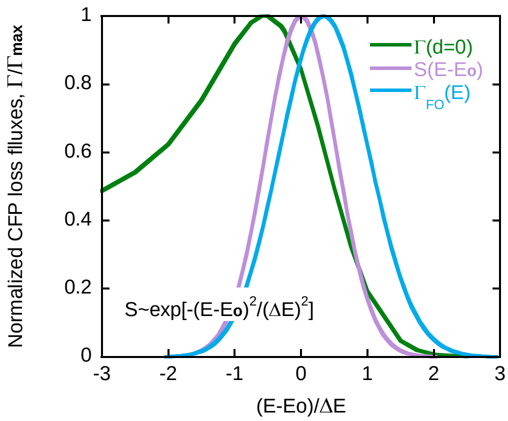

Fig. 4: Fluxes of diffusive (green) and first orbit (blue) losses of CFPs vs energy in case of a Gaussian source with ΔE = 0.39E_0.

It is seen that, contrary to the diffusive loss of slightly thermalized CFPs, the FO losses are shifted to higher energies.

3. Conclusions

The simultaneous consideration of both convective and diffusive transport has been shown to be important in fast ion simulations, as they essentially affect the radial profiles of partly thermalized charged fusion products at the plasma edge. In addition to diffusion, an outward radial convection can significantly enhance the CFP loss from tokamak plasmas. Further, it modifies the energy spectra of lost fast ions as well.

In general, though sometimes not considered in fast ion modelling, the convective radial transport will significantly influence the confinement of slightly thermalized charged fusion products and should be taken into account, especially when anomalous loss of fusion products is to be interpreted [4,5].

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