Laser wakefield acceleration in corrugated plasma channel

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One of the most important areas where the laser dynamics plays a crucial role is in the laser wakefield accelerators. Here we show that plasma inhomogeneities, specifically corrugated plasma channels, can be used to modify the laser trajectory and hence the trajectories of the trapped electrons in the wake of the laser. As a result the radiation signatures of the trapped electron bunch can be modified. For this purpose, we have performed two dimensional (2D) particle-in-cell (PIC) simulations using OSIRIS[1], and post processed the tracks of the accelerated electrons with jRad [2], which is a massively parallel post-processing radiation diagnostic that takes the track information from 3D/2D PIC simulations and determines the full radiation spectrum of the corresponding particle(s).

We first begin with obtaining the equation for transverse coordinates of the laser centroid in a corrugated plasma channel, resorting to variational principle approach followed by Duda *et al.* [3]. In the case of inhomogeneous plasma with density $n = n_0 f(\vec{r}_\perp)$, where $f(\vec{r}_\perp)$ is inhomogeneity function of general form and depends upon transverse coordinates $\vec{\mathrm{r}}_\perp \equiv (\mathrm{x},\mathrm{y})$, the Lagrangian corresponding to the paraxial wave equation of a linearly polarized laser can be given as

$$\mathfrak{L} \;=\; \nabla_\perp a \cdot \nabla_\perp a^* - ik_0(a\partial_\tau a^* - a^*\partial_\tau a) + f(\vec{r_\perp})aa^*. \tag{1}$$

where $a \ll 1$ is the normalized vector potential, normalized with $e/(mc^2)$, m and e are the electron mass and charge, c is the speed of light, k_0 is the wavenumber of the laser and $\omega_p = \sqrt{n_0 e^2/(m\varepsilon_0)}$ is the plasma frequency, with ε_0 defined as the dielectric constant in vacuum. We normalize all the time-unit-variables with ω_p^{-1}, and space-unit-variables with inverse of plasma wavenumber $k_p^{-1} \equiv c/\omega_p$. $\psi = t - z$, $\tau = z$ are the coordinates in the frame of reference of the laser, and the Lagrangian is obtained in slowly varying envelope approximation regime $|\partial_\tau a| << k_0|a|$, neglecting the plasma wave effects. It should be noted that by solving the Euler-Lagrange equation[4], $\partial_X \mathfrak{L} - \partial_\tau(\partial_{\partial_\tau X}\mathfrak{L}) - \partial_\psi(\partial_{\partial_\psi X}\mathfrak{L}) = 0$, for $X \equiv a^*$, we retrace the wave equation for the laser amplitude. We use the trial function for the vector potential $a = A\left[ik_x(x-x_a)\right] \times \exp\left[-(1-\alpha_x)(x-x_a)^2/W_{xa}^2\right] \times \exp\left[-(1-\alpha_y)y^2/W_{ya}^2\right]$[3], where $A = \sqrt{\xi}\,\exp[i\chi]$ is the complex amplitude, x_a is the transverse centroid coordinates of the vector potentials along x axis. W_{xa}, W_{ya} represents the spot-sizes, α_x and α_y are related to vector potential curvatures in x and y direction respectively, and k_x is the transverse wavenumber. Substituting the trial function in Eq. 1, we get the Lagrangian in terms of laser parameters (A, Φ, W_{xa}, x_a, k_x, etc.) which now are the new dependent variables and functions of independent coordinates ψ and τ. Upon integrating over the transverse coordinates $(x,\ y)$, we get the reduced Lagrangian $\langle\mathfrak{L}\rangle = \int \mathfrak{L} d\vec{r}_\perp$. Then, following Duda *et al.* [3], we solve the Euler-Lagrange equations[4] for $X \equiv \chi$, k_x, α_x and α_y, to get $\partial_\tau P = 0$ (Power conservation), $k_x = -k_0\partial_\tau x_a$, and $\sigma_{x,y} = -\frac{k_0}{4}\partial_\tau(W_{xa}^2, W_{ya}^2)$, where $P = A^2 W_{xa} W_{ya}$. After substituting these equations, we further deduce $\langle\mathfrak{L}\rangle$ as

$$\langle\mathfrak{L}\rangle = \frac{1}{2W_{xa}^2} + \frac{1}{2W_{ya}^2} - \frac{k_0^2}{8}(\partial_\tau x_a)^2 + \frac{I_n^0(x_a, W_{xa}, W_{ya})}{2W_{xa}W_{ya}}. \quad (2)$$

where $I_n^i = \frac{2}{\pi}\int\int [f(x,y)-1](x-x_a)^i \exp\left[-2\frac{(x-x_a)^2}{W_{xa}^2} - 2\frac{y^2}{W_{ya}^2}\right]$ dx dy is the contribution from the plasma inhomogeneities and i is an integer. Considering a non evolving laser in ψ with constant spot size $W_{xa} = W_{ya} = W_0$ and solving the Euler-Lagrange equation for $X \equiv x_a$, we get the equation for x_a as,

$$\partial_\tau^2 x_a + \frac{2}{k_0^2 W_0^4} I_n^1 = 0. \quad (3)$$

For a parabolic plasma channel corrugated with frequency $\omega_{\rm corr}$, channel radius $W_{\rm ch}$ and density profile $f(x) = 1 + \frac{1}{W_{\rm ch}^2}\left[x - x_{\rm c0} - {\rm A_{corr}}\,\sin(\omega_{\rm corr}\tau)\right]^2$, where $\rm x_{c0}$ is the axis of the channel at $\tau = 0$, $A_{\rm corr}$ is the amplitude of channel oscillation; the drift in the transverse laser centroid with respect to $x_{\rm c0}$ can be given as

$$\Delta x_0 = x_a - x_{\rm c0} = \Delta x_0 \cos(\omega_{\rm ch}\tau) - \frac{A_{\rm corr}}{R^2-1}\left[\sin(\omega_{\rm corr}\tau) - R\tau\,\sin(\omega_{ch}\tau)\right], \quad (4)$$

where $\Delta x = x_{a0} - x_{\rm c0}$, $R = \omega_{\rm corr}/\omega_{\rm ch}$ and $\omega_{\rm ch} = 1/(k_0 W_{\rm ch})$ is the channel frequency corresponding to the plasma channel depth. When $\omega_{ch} = \omega_{corr}$ and $\Delta x_0 = 0$, the laser transverse modulation amplitude increases with time in each cycle as

$$\triangle x = -\frac{A_{\rm corr}}{2}\left[\omega_{\rm corr}\tau\cos(\omega_{\rm corr}\tau) - \sin(\omega_{\rm corr}\tau)\right] \quad (5)$$

To demonstrate the modulation of laser, and thus the trapped electron bunch in the corrugated plasma channel, we have performed 2D PIC simulations. We use a moving window that propagates with velocity c, has longitudinal length $15c/\omega_p$ and transverse width $80c/\omega_p$, divided into 1500 and 800 cells in the respective directions, with 9 particles per cell. The length of the plasma is $2000c/\omega_p$, and the ions form an immobile neutralizing fluid background. We consider a scenario where a Gaussian laser pulse with normalized peak amplitude $a_0 = 1.0$, pulse length $L = 3c/\omega_p$, spot-size $W_0 = 6c/\omega_p$ and frequency $\omega_0 = 20\omega_p$ is propagating in a

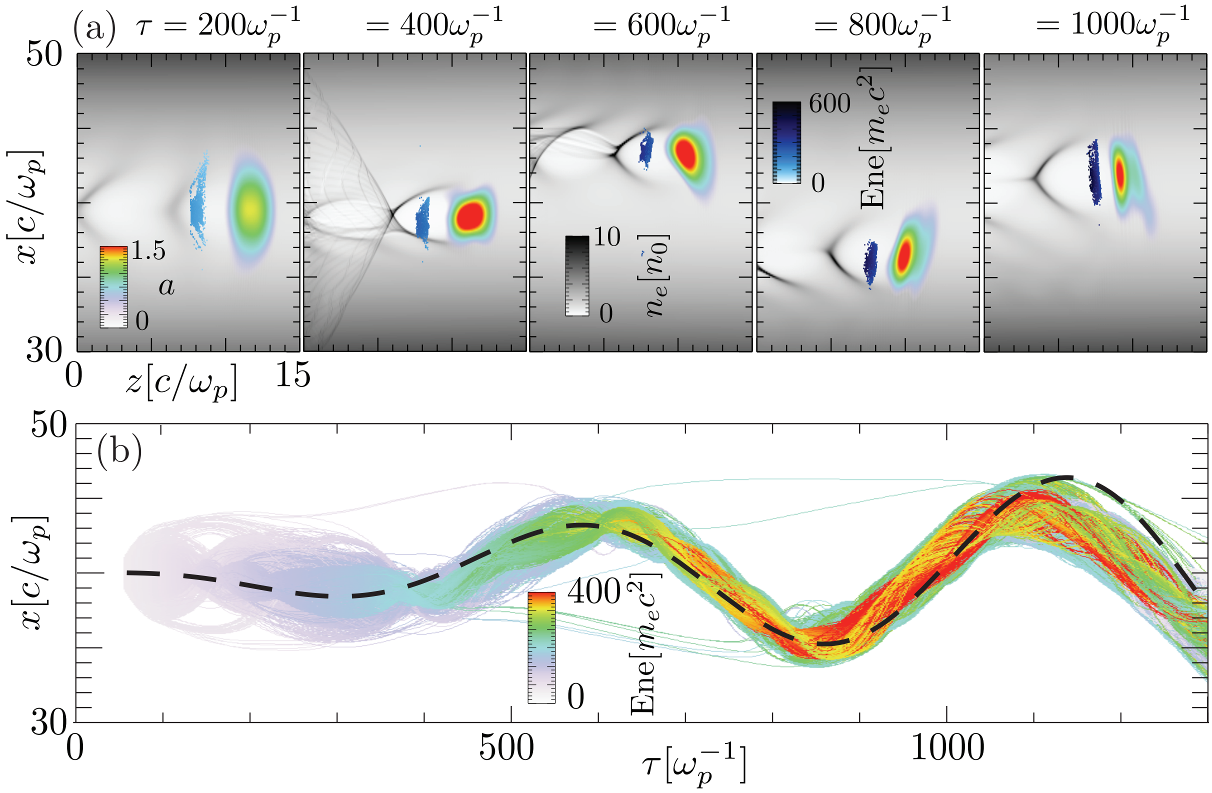

Figure 1: Laser propagation in corrugated plasma channel ($\omega_{\text{corr}} = \omega_{\text{ch}}$) :(a) simulation snapshots of vector potential envelope of the laser, background plasma density and the trapped externally injected electrons in the x-z plane at time $\omega_p\tau = 200, 400, 600, 800$ and 1000. The laser in initialized on axis ($x_a0 = x_{c0}$). (b) tracks of 500 trapped particles with color scheme representing their energies. The dashed line is the trajectory of the laser predicted by the theoretical model (Eq. 5).

corrugated plasma channel with $A_{\text{corr}} = 1c/\omega_p$ and $\omega_{\text{corr}} = \omega_{ch} = 0.01\omega_p$. The laser pulse is initialized on axis ($\Delta x_0 = 0$), thus we expect laser to follow Eq. 5. An external test electron bunch with initial longitudinal momentum $p_z = 20mc$ and beam density $n_{0b} = 10^{-5}n_{0p}$ is initialized in the box such that these electrons can catch the nonlinear wake (bubble) behind the laser. In Fig. 1(a) we plot the snapshots of the laser envelope in x-z plane at various times, showing the oscillation of the laser pulse (in accordance with the Eq. 5), the bubble and hence the trapped electron bunch in the corrugated plasma channel. The trajectories of 500 trapped particles are plotted in Fig.1(b) which is a combination of betatron oscillation of the bunch in the bubble as well as the oscillation of the bubble itself due to the laser dynamics. The color of the tracks represents the energy, with peak energy at 250MeV. The dashed line is the theoretical estimation (Eq. 5) for the laser trajectory, suggesting that by changing the corrugated plasma channel parameters the dynamics of the laser and hence the trajectories of the trapped particles can be modified, which may lead to distinct radiation characteristics of the electron bunch as compare to the conventional betatron radiation. We post processed the tracks of randomly selected 500 trapped particles with jRad [2], and the radiation properties are plotted in the Fig.2. In Fig. 2(a) we plot the spectrum of radiation emitted by the trapped particles along the x axis at $y = 0$, with the position of the spectrometer at $z = 5500c/\omega_p$, and Fig. 2(b) shows the radiation energy distribution (in arbitrary units) in the x-y plane. Well separated two peaks at $x = -100c/\omega_p$ and $x = 175c/\omega_p$ is due to the higher modulation amplitudes $\sim 10c/\omega_p$ of the electron trajectories and the asymmetry in the spectrum is caused by the combination of different peak-numbers of electron-trajectory modulations in x axis and particle energy (2 peaks in -x and 1 peak in + x direction with majority of particle with energy $\sim$ 200MeV).

In conclusion, we have demonstrated that a corrugated plasma channel can be used to produce electron bunch in LWFA with unconventional trajectories, leading to distinct radiation characteristics.

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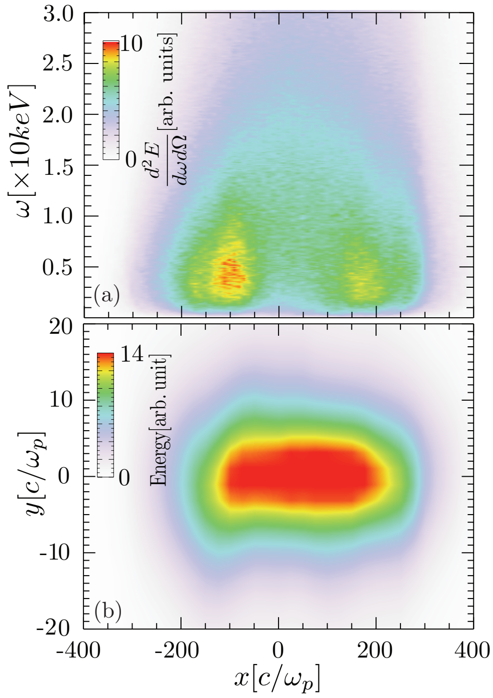

Figure 2: Spectrum of radiation emitted by the arbitrarily chosen 500 trapped accelerating particles: (a) radiation frequency distribution on x axis (y=0), and (b) radiated energy distribution in x-y plane. The spectrum is obtained at $z = 5500c/\omega_p$.

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