Effect of wall thickness and plasma rotation on RWM

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Abstract
In this paper we introduce a semianalytical model in cylindrical geometry to study Resistive Wall Modes stability including thick wall effects, plasma rotation and additional energy dissipation.

Introduction
Resistive Wall Modes (RWM) are MHD instabilities (low-n external kink) whose growth rate is determined by the interaction of the mode itself with the surrounding conducting structures [1]. Indeed, if the conducting wall is close enough, any plasma movement induces eddy currents which counteract the instability itself. This brings the time scale from the very fast Alfvénic time scale (typically microseconds) to the much slower electromagnetic times. An usual simplification [1] consists of assuming the mode so slow that the corresponding current density perfectly penetrates the wall - the so-called "thin wall approximation". In advanced tokamak scenarios the mode growth rate can be so high that this assumption may not be valid because of the skin effect [2] - e.g. the steady-state advanced scenario in ITER aims at high plasma pressure exceeding the RWM stability limit [3]. Recently, several analytical [2] and numerical [4] computations have demonstrated that, for non-rotating modes, the proper inclusion of wall thickness in the analysis results in substantially higher growth rates than with the thin-wall approximation - this is due, among other reasons, to the fact that only a fraction of the wall contributes to stabilization, due to skin effect. Very recent studies [5],[7] suggest that the effect of wall thickness may be even more noticeable in presence of plasma rotation. Aim of this paper is to present a semianalytical model aimed at verifying this conjecture.

Formulation
From the energy principle, the non-linear RWM dispersion relation can be written as [6]:

\[(p + in\Omega)^2K + (p + in\Omega)D + \delta W_F + \delta W_{EM}(p) = 0 \quad (1)\]

where \(p = \gamma + i\omega\) is the growth rate, such that all the quantities have a time dependence \(e^{\gamma t}\), \(K\) is the kinetic integral of the plasma inertia, \(\delta W_F\) is the perturbed fluid potential energy of the plasma, \(\delta W_{EM}(p)\) is the perturbed electromagnetic energy, \(\Omega\) is the plasma fluid rotation frequency, \(D\) is related to damping effects and \(n\) is the toroidal mode number.

We consider a cylindrical plasma of length \(L = 2\pi R_0\) and radius \(r = 1\), obtained from torus of aspect ratio \(R_0\). The coordinate system is cylindrical \((r, \theta, z)\) in which \(z\) is the axis of cylindrical plasma. We consider a flat current density \(J = J_0\hat{z}\) and equilibrium magnetic field \(B = B_0\hat{z} + B_0\hat{\theta}\), where \(B_0(\theta) = \mu_0 J_0 r/2\) with \(|r| \leq 1\). The plasma density and \(q\) profiles are \(\rho(r) = \rho_0\) and \(q(r) = 2B_0/(\mu_0 R_0 J_0)\), respectively.

In this geometry, the terms \(K\) and \(\delta W_F\) become [6]:

\[K = 2\pi^2 R_0^2 r_A^2 \frac{\mu}{\mu_0} \frac{\rho_0^2}{(m + nq_0)^2} \quad (2)\]
\[
\delta W_F = 4\pi^2 R_0 \frac{\mu}{\mu_0} \left( \frac{1}{2} - \frac{v}{m + n q_0} \right)
\]

where \( m \) is the poloidal mode number, \( \mu = |m|, v = m/\mu \) and \( \tau_A \) is Alfvén time.

We compute the term \( \delta W_{EM} = \frac{1}{2\mu_0} \int_{V_{ext}} |B_1|^2 dV \), with \( V_{ext} \) volume external of plasma, in the cylindrical limit \( (R_0 \to \infty) \), introducing the magnetic flux function \( \phi_1(r, \theta, z) \), such that \( B_1 = \nabla \phi_1 \hat{z} \), factorized as \( \phi_1(r, \theta, z) = \phi(r) e^{im\theta + \frac{m q_0}{\mu_0} z} \).

Several assumptions are made on the region outside the plasma, for the computation of \( \delta W_{EM} \).

- **Without any wall**
  
  In this case, the equation is
  
  \[ V_2^2 \phi_1 = 0 \]  

  with (normalized) boundary condition \( \phi(r = 1) = 1 \). The solution of (4) is
  
  \( \phi(r) = r^{-\mu}(r > 1), \quad \delta W_{EM}(p) = \delta W_{\infty}^{b} = 4\pi^2 R_0 \frac{\mu}{\mu_0} \frac{1}{2} \)  

- **Ideal thin wall**

  An ideally conducting cylindrical wall of radius \( r = b \) introduces an additional boundary condition \( \phi(r = b) = 0 \), so that the solution of (4) outside the plasma \( (1 < r < b) \) becomes:
  
  \( \phi(r) = \frac{1}{1 - b^{-2\mu}} r^{-\mu} + \frac{1}{1 - b^{+2\mu}} r^{+\mu}, \quad \delta W_{EM}(p) = \delta W_{\infty}^{b} = \delta W_{\infty}^{b} \frac{1 + b^{-2\mu}}{1 - b^{-2\mu}} \)  

- **Thin resistive wall**

  Considering a wall with a finite conductivity \( \sigma \) and a thickness \( d \), we introduce the wall penetration time \( \tau_w = \mu_0 \sigma bd \). Assuming \( d \ll b \) (thin wall approximation), the boundary conditions become [6]:

  \[ \left\{ \begin{array}{lcr}
  \phi(1) = 1 \\
  \phi(b^-) = \phi([b + d]^+) \\
  \frac{b \phi'(b^+) - \phi'(b^-)}{\phi(b^+)} = 2p \tau_w
  \end{array} \right. \]  

  and the solution of (4) outside the plasma \( (r > 1) \) is:

  \[ \left\{ \begin{array}{lcr}
  \phi(r) = \frac{1}{1 + \frac{\mu}{1 + \frac{1}{\mu_0} d}} r^{-\mu} + \frac{1}{1 - \frac{\mu}{1 + \frac{1}{\mu_0} d}} r^{+\mu} & r \leq b \\
  \phi(r) = \frac{1}{1 + \frac{\mu}{1 + \frac{1}{\mu_0} d}} r^{-\mu} & r > b
  \end{array} \right. \]  

  with \( \tau_w' = \tau_w \frac{1 - b^{-2\mu}}{\mu} \). Since the conductivity is finite, in this case it is necessary to add the energy dissipation in the wall to the electromagnetic energy:

  \[ \delta W_i = \frac{1}{2p} \int_{V_{ext}} \eta |J|^2 dV \]  

  so that in the end [6]:

  \[ \delta W_{EM}^{thin}(p) = \frac{\delta W_{\infty}^{b}}{1 + p \tau_w'} \]
Thick resistive wall

In this case, the equations to be solved are (4) in vacuum, while in the wall we have:

\[-\nabla^2 T \phi_1 = \mu_0 \sigma \rho \phi_1\]  

(11)

with boundary conditions:

\[
\begin{align*}
\phi(1) &= 1 \\
\phi(b^-) &= \phi(b^+) \\
\phi'(b^-) &= \phi'(b^+) \\
\phi([b + d]^-) &= \phi([b + d]^+) \\
\phi'([b + d]^-) &= \phi'([b + d]^+) 
\end{align*}
\]  

(12)

which provide (7) for \(d \ll b\).

Equation (11) is a modified Bessel equation of order \(\mu = |m|\), whose solution \((b < r < b + d)\) is:

\[
\phi_{\text{wall}}(r) = c_{w1} K_\mu(\rho) + c_{w2} I_\mu(\rho), \quad \rho = r \sqrt{\mu_0 \sigma \rho}
\]  

(13)

where the values of the constants are determined numerically by imposing (12). Also (9) is computed numerically.

Result

We choose the following parameters: \(B_0 = 3.5, R_0 = 5, m = 2, n = -1, \mu_0 = 1.25e-6, J_0 = 1e6, \sigma = 1e6, \rho_0 = 5.8e-6\).

First of all, we analyse the case with no rotation. Fig. 1 refers to a case when the skin depth \(s \ll d\), so that the thin wall approximation is valid, since the normal component of magnetic perturbations is constant across the wall. In this case, the results of (13) and of (8) are practically coincident, as expected. Conversely, if \(s/d \approx 1\) we have(Fig. 2), the thin wall approximation is significantly different from the reference thick solution, so that we have \(\forall \ p \ \gamma_{\text{thick}}(p) > \gamma_{\text{thin}}(p)\). These results are in perfect agreement with [4].

![Figure 1: Thin wall limit, s/d ≈ 60](attachment:figure1.png)
Including a non-vanishing rotation velocity (Fig. 3), apparently the plasma rotation only is not able to stabilize the mode in the explored range, if the plasma is assumed ideal. Conversely, inclusion of a damping coefficient can stabilize the mode both with thick and thin wall. In this case, the thick wall is clearly favourable in the stabilization of the mode.

**Conclusion**

We analysed the Resistive Wall Modes in cylindrical geometry and carried out the exact solution of the nonlinear RWM dispersion equation, to treat both thin and thick resistive walls. Without rotation, the usual thin wall approximation produces an underestimation of the growth rate. Conversely, considering the plasma rotation, the thick wall may improve stabilization of the mode, although in the explored range a stabilization may be reached only with the inclusion of an additional damping from the plasma.

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**References**


