On radio detection of EAS from High Energy Cosmic Rays

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Radio pulses associated with Extended Air Showers (EAS) produced in terrestrial atmosphere by High Energy Cosmic Rays (UHECR) of energy $10^{17}$ eV and above, are now routinely observed by dedicated radio instruments on ground. This may offer a new way for elucidating the nature and origin of the involved primary particle, an open question still not solved.

A mandatory intermediate step will be the understanding of involved radio emission processes. At very first order, EAS secondary particles can be described as an electrical charges packet moving down to the ground at relativistic speed. Two basic processes (Scholten 2008) are then able to produce significant, impulsive change in the electric field measured by a distant observer: a “charge excess” due to differential absorption of negatively and positively charged particles along the EAS development; a “transverse current”, due to the Lorentz force exerted by Earth’s magnetic field (Kahn and Lerche 1966).

In order to obtain a rough analytical estimate, in terms of intensity, spectrum and polarisation, of the radio signal observable on ground with an array of radio antennas, one can consider the shower as a localized charge $\eta Q$ moving towards the ground through the atmosphere of constant refractive index $n$. Its velocity $\vec{v} = \beta \vec{c}$ with Lorentz factor $\gamma = (1 - \beta^2)^{-1/2} \gg 1$ (typically $\gamma \sim 40$) is that of the bulk of charged particles in the shower. $Q$ is the total charge of secondary electrons and $\eta$ (typically $\eta = 0.2$) defines the “charge excess” of electrons over positrons. The corresponding field potentials at space-time point $(\vec{r}, t)$ are the standard Lienard-Wiechert potentials:

$$\Phi_{\text{exc}}(\vec{r}, t) = \frac{Q_{\text{ret}}}{4\pi \epsilon_0 c^2} \left[ (1 - n\beta \cdot \vec{u}) R \right]_{\text{ret}}^{-1}$$

$$\vec{A}_{\text{exc}}(\vec{r}, t) = n^2 \beta \Phi(\vec{r}, t) / c$$

(1)

$R \vec{u}$ is the vector from the moving charge to the antenna and the subscript “ret” indicates that the quantities inside the square brackets are to be evaluated at observer’s retarded time $t_{\text{ret}} = t - nR(t_{\text{ret}}) / c$.

In the same way, the field potential of the moving current $\vec{J}$ due to the acceleration by the Lorentz force associated with the Earth’s magnetic field will be:

$$\vec{A}_{\text{sep}}(\vec{r}, t) = \frac{\vec{J}_{\text{ret}}}{4\pi \epsilon_0 c^2} \left[ (1 - n\beta \cdot \vec{u}) R \right]_{\text{ret}}^{-1}$$

(2)
In both cases, two versions of the resulting field can be produced, depending on whether the charge is moving slower or faster than the velocity of light in the medium, that is \( n\beta < 1 \) or \( n\beta > 1 \). In the first version, - the infra-luminous case - , only one instant in the charge’s past history has a light cone which reaches a given location in space-time \((\vec{r}, t)\) (e.g. an antenna, at a given observer’s time, looking at one retarded altitude). In the second version, - the supra-luminous case - , the field visibility is restricted to from inside the Čerenkov cone of half angle \( \sin^{-1}(1/n\beta) \). Since the air index is variable with altitude and composition (in particular humidity) around \( n \approx 1.003 \), the two infra and supra luminous cases of each emission mechanism can coexist within a single shower observed by a spatially distributed antenna array.

For characterizing energetic secondary particles (parameter \( \gamma \) in \( n = 1 \) case), relevant parameters are \( \gamma' = (1 - n^2\beta^2)^{-1/2} \) and \( \gamma_c = (n^2\beta^2 - 1)^{-1/2} \) for no-Čerenkov and Čerenkov cases respectively.

Expressions of the field spectrum can be easily derived (Meyer-Vernet 2008) by Fourier transforming equations (1) and (2) and their derivatives. For the charge excess mechanism and both infra and supra luminous cases, the main field will be polarized along the radial direction \( \vec{\delta} = \vec{d}/d \), where \( \vec{d} \) is the vector connecting the antenna to closest approach point of the moving charge:

\[
\vec{E}_{\text{exc}}(\vec{r}, \omega) = \frac{Q}{2\pi c_0 n^2 \gamma c\omega} \frac{\omega d}{\sqrt{\gamma v}} K_1\left(\frac{\omega d}{\sqrt{\gamma v}}\right) e^{i\omega x/v}\vec{\delta} \\
\vec{E}^\gamma_{\text{exc}}(\vec{r}, \omega) = \frac{i Q}{4\pi c_0 n^2 \gamma c\omega} \frac{\omega d}{\sqrt{\gamma c v}} H^1_1\left(\frac{\omega d}{\sqrt{\gamma c v}}\right) e^{i\omega x/v}\vec{\delta}
\]

For the charge separation mechanism, the field will be parallel to the transverse current \( \vec{J} \) aligned with the \( \vec{v} \times \vec{B} \) vector, where \( \vec{B} \) is the local geomagnetic field.

\[
\vec{E}_{\text{sep}}(\vec{r}, \omega) = \frac{i \vec{J}}{2\pi c_0 c^2 v} \omega K_0\left(\frac{\omega d}{\sqrt{\gamma v}}\right) e^{i\omega x/v}\vec{\delta} \\
\vec{E}^\gamma_{\text{sep}}(\vec{r}, \omega) = -\frac{i \vec{J}}{4\pi c_0 c^2 v} \omega H^1_0\left(\frac{\omega d}{\sqrt{\gamma c v}}\right) e^{i\omega x/v}\vec{\delta}
\]

In equations (3) to (6), \( K_0, K_1, H^1_0, H^1_1 \) are modified Bessel functions and Hankel functions of third kind respectively. For large arguments (that is at high frequency or large distance from the shower core) the former behave as nearly exponentially decreasing functions, a behaviour which is well observed in real data (cf. Fig.1), whereas the Hankel functions have an oscillating behaviour. In order to compare relative efficiency of various mechanisms and predictions with
observations, one has to evaluate the current $\tilde{J}$. Following Allan (Allan 1971), $\tilde{J} \approx 2QL/\Delta t$ where $L$ is the mean displacement due to Lorentz force acting during the time $\Delta t = L_0/\rho c$ ($L_0 =$ radiation length, $\rho =$ air density), so that:

$$\tilde{J} \approx \left( 2Qe^2 \frac{\omega_B L}{\gamma' \rho c} \sin \alpha \right) \tilde{w} \quad \tilde{w} = \frac{\tilde{v} \times \tilde{B}}{||\tilde{v} \times \tilde{B}||}$$

(7)

in which $\omega_B$ is the electron gyro frequency and $\alpha$ the angle ($\tilde{v}, \tilde{B}$) between the geomagnetic field and the direction of motion (the shower axis).

Introducing relationship between total charge and primary energy $W_p$ by $Q/e \approx 8 \times 10^7 (W_p/10^{17})$ (Abu-Zayyad, Belov, and Bird 2001), fields (3) to (6) take the approximated numerical forms, expressed in V/m/Hz:

$$E_{exc}^{d} (\tilde{r}, \omega) = 7.7 \times 10^{-10} \eta (W_p/10^{17}) \frac{1}{\gamma' \rho} \frac{\text{d} \chi}{\gamma' \rho} \frac{K_0 \left( \frac{\text{d} \chi}{\gamma' \rho} \right)}{\text{d} \gamma' \rho} e^{i\omega \gamma' \rho} \tilde{w}$$

(8)

$$E_{exc}^{c} (\tilde{r}, \omega) = 1.2 \times 10^{-9} \eta (W_p/10^{17}) \frac{1}{\gamma' \rho} \frac{\text{d} \chi}{\gamma' \rho} \frac{H_1 \left( \frac{\text{d} \chi}{\gamma' \rho} \right)}{\text{d} \gamma' \rho} e^{i\omega \gamma' \rho} \tilde{w}$$

(9)

Hence the field amplitude ratios:

$$\frac{|E_{sep}|}{|E_{exc}|} = \mathcal{R} \left( \frac{K_0 \left( \frac{\text{d} \chi}{\gamma' \rho} \right)}{K_1 \left( \frac{\text{d} \chi}{\gamma' \rho} \right)} \right) \quad \mathcal{R} = \mathcal{R}_c \approx \frac{14 \sin \alpha}{\eta (\rho/\rho_{ground})}$$

$$\frac{|E_{sep}|}{|E_{exc}|} = 1.6 \left( \frac{H_1 \left( \frac{\text{d} \chi}{\gamma' \rho} \right)}{K_1 \left( \frac{\text{d} \chi}{\gamma' \rho} \right)} \right)$$

One can see that “transverse current” mechanism is much more efficient than “charge excess” one (by about 1 or 2 orders of magnitude, especially from the shower part at highest altitude for which $\rho/\rho_{ground} \ll 1$), excepted when shower axis is closely aligned with geomagnetic field direction ($\sin \alpha \ll 1$). At same intensity level, in addition to being quite differently polarised, the two mechanisms will differ by slightly different spectral slopes in the higher frequency part of the spectrum. Čerenkov and no-Čerenkov emission are of comparable amplitude, except at large frequencies or distances for which the Čerenkov mechanism dominates, but the Čerenkov spectrum might be recognized by measurable intensity modulations over the antenna array (due to the phase term in complex Hankel functions).

The Nançay CODALEMA research instrument (Ardouin 2009) is made, in its present configuration, of 67 sensitive cross polarized active dipoles covering an area of 1.5 km $\times$ 1.5 km and triggered by a surface detector (13 plastic scintillators) covering an area of 0.3 km $\times$ 0.3 km.
Two illustrating examples are given below. In Fig.1 (left panel), the single antenna, raw power spectrum of an EAS radio event is displayed (black line) over the galactic background (red line). Note the presence of strong RFI in AM and FM broadcasting bands. In the right panel, the corresponding calibrated spectrum (in black) is shown together with theoretical curve (in red) from Eq.9 for a shower of primary energy $4 \times 10^{17}$ eV, seen at 100m distance, with $\gamma = 40$ and $n = 1.00026$ ($\gamma' = 100$).

The second example displays the observed relationship between polarisation of radio signal measured by CODALEMA and the direction of $\mathbf{v} \times \mathbf{B}$ vector, showing pre-eminence of “transverse current” mechanism as expected from Eq.8-9. Only a very few events (or isolated antenna), in a proportion of ~5%, might be “charge excess” candidates.

References: