Explosive Ballooning Mode Instability in Tokamaks: Modelling the ELM Cycle

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1. Introduction

There exists two main operational regimes in a tokamak fusion device: the low-confinement mode (L-Mode) and the high-confinement mode (H-Mode). The H-Mode is distinguished from the L-Mode through a steep pressure gradient region at the edge, called the pedestal (Fig. 1). Fusion devices will likely operate in H-Mode to optimise fusion performance. However the steep edge pressure gradient usually triggers Edge Localised Modes (ELMs). These are quasi-periodic instabilities which have a filamentary structure, and grow very rapidly. They release a large amount of energy and particles which can erode components on future fusion devices, such as ITER. Therefore it is very important to understand ELMs and their associated energy.

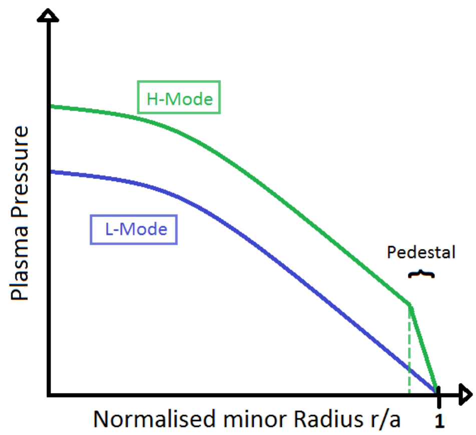


Figure 1: *Schematic pressure profile for L- and H-Mode*

2. Analytical Calculations

We employ the Clebsch coordinate system which defines the equilibrium magnetic field as $B_0 = \nabla\psi \times \nabla\alpha$ where ψ labels the flux surfaces, and therefore represents a radial component, and α labels the field lines on the flux surfaces.

Wilson and Cowley, [1], extended earlier calculations, [2, 3, 4], to tokamak geometry and derived the following ballooning mode envelope equation from non-linear MHD which describes the displacement ξ of a field-aligned flux tube:

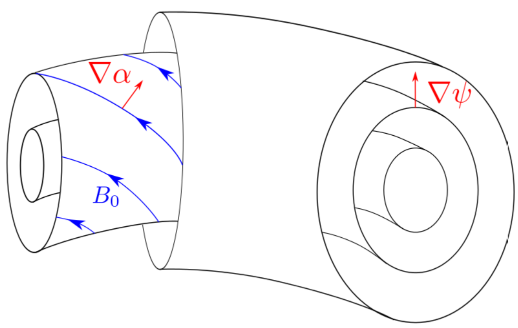


Figure 2: *Spatial components of the coordinate system where $B_0 = \nabla\psi \times \nabla\alpha$*

$$\underbrace{\frac{\partial}{\partial\alpha}\frac{\partial^2\xi}{\partial t^2}}_{\text{Inertia Term}} = \underbrace{(\delta - \frac{(\psi-\psi_0)^2}{\Delta^2})\frac{\partial\xi}{\partial\alpha}}_{\text{Linear Instability Drive}} - \underbrace{\frac{\partial^2 u}{\partial\psi^2}}_{\text{Field Line Stability Term}} + \underbrace{\frac{\partial\xi^2}{\partial\alpha}}_{\text{Non-linear Growth Drive}} + \underbrace{\frac{\partial\xi}{\partial\alpha}\frac{\partial^2\overline{\xi^2}}{\partial\psi^2}}_{\text{Quasi-Linear-Nonlinearity Term}} + \underbrace{\nu_\alpha\frac{\partial^3}{\partial\alpha^3}\frac{\partial\xi}{\partial t}}_{\text{Viscosity Term}} \quad (1)$$

where $\xi = \frac{\partial u}{\partial\alpha}$ and an overbar means an average over α.

Coefficients that depend on the equilibrium and leading order linear ballooning mode solution have been absorbed into re-scaled variables so that we can investigate the generic ballooning mode envelope equation. The pre-factor of the linear instability drive term represents how close the system is to marginal stability, assuming a maximum in the local growth rate at $\psi \sim \psi_0$. We can express this variable in terms of the pressure gradient: $\delta = \frac{p'_0 - p'_c}{p'_c}$ where p'_0 is the pressure gradient and p'_c is the critical pressure gradient for ballooning instability, so $\delta = 0$ at marginal stability.

3. Results and Discussion

Evolution

For a first investigation we perform a simple balancing of the non-linear terms of the ballooning mode envelope equation which leads to the following relations:

$$\xi \propto (t_0 - t)^{-2} \qquad \frac{\Delta\psi^2}{\Delta\alpha} \propto \xi \qquad \Delta\alpha \propto (t_0 - t)^{0.5} \qquad \Delta\psi \propto (t_0 - t)^{-0.75} \quad (2)$$

where $\Delta\alpha$ is the width of the filament in the $\nabla\alpha$ direction and $\Delta\psi$ is the width of the filament in the $\nabla\psi$ direction; t_0 is a quantity that depends on initial conditions. The last two relations are derived using the viscosity term.

Indices, p, of the asymptotic form $\sim (t_0 - t)^p$

Summary of Results	p_ξ	p_α	p_ψ	$\frac{(2p_\psi - p_\alpha)}{p_\xi}$
Theory no viscosity	-2.	x	x	1
Simulation no viscosity	-2.62	1.51	-0.48	0.94
Theory with viscosity	-2.	0.5	-0.75	1
Simulation ($\nu_\alpha = 0.01$)	-3.16	2.21	-0.53	1.03

The predicted indices (see table) indicate that the width $\Delta\alpha$ is shrinking and the width $\Delta\psi$ is growing as the finite time singularity, $t \to t_0$, is approached. Numerical simulations confirm this (Fig. 3).

The results always exhibit an explosive behaviour of the filaments independent of whether or not viscosity is included. However, the simple balancing of terms does not give accurate quantitative results for the individual indices p_ξ, p_α and p_ψ, (see table). Nevertheless, the relation $\frac{(2p_\psi - p_\alpha)}{p_\xi} \approx 1$, which comes from balancing the non-linear growth drive term and the quasi-linear nonlinearity term, is very robust, agreeing well with simulations with and without viscosity. This indicates that these two terms are dominating the evolution close to the finite time singularity.

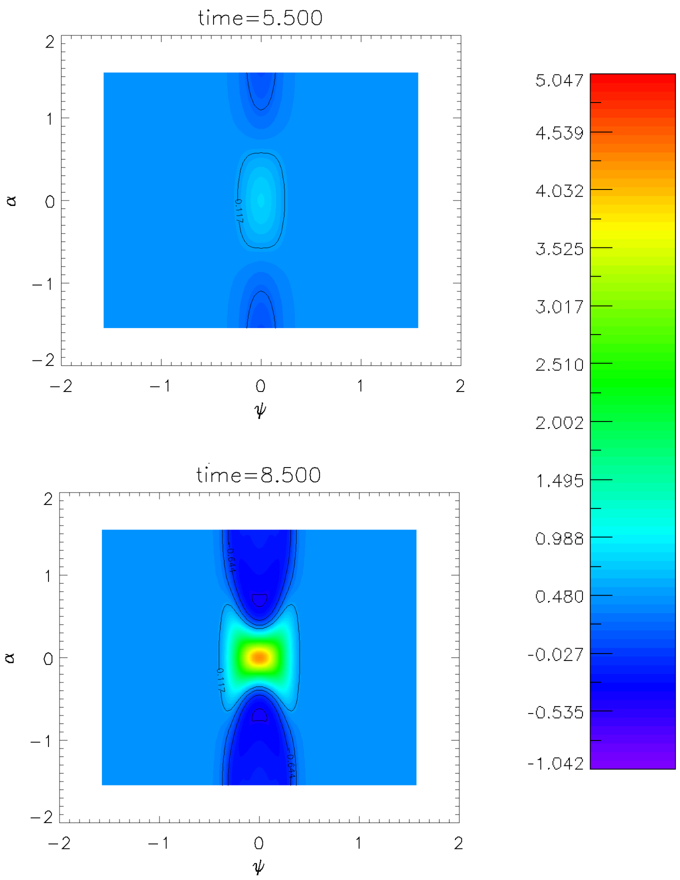


Figure 3: *Top: Contour plot of the displacement ξ when nonlinear behaviour starts to dominate. Bottom: Contour plot deeper into the nonlinear region: ξ widens in ψ and narrows in the α-direction.*

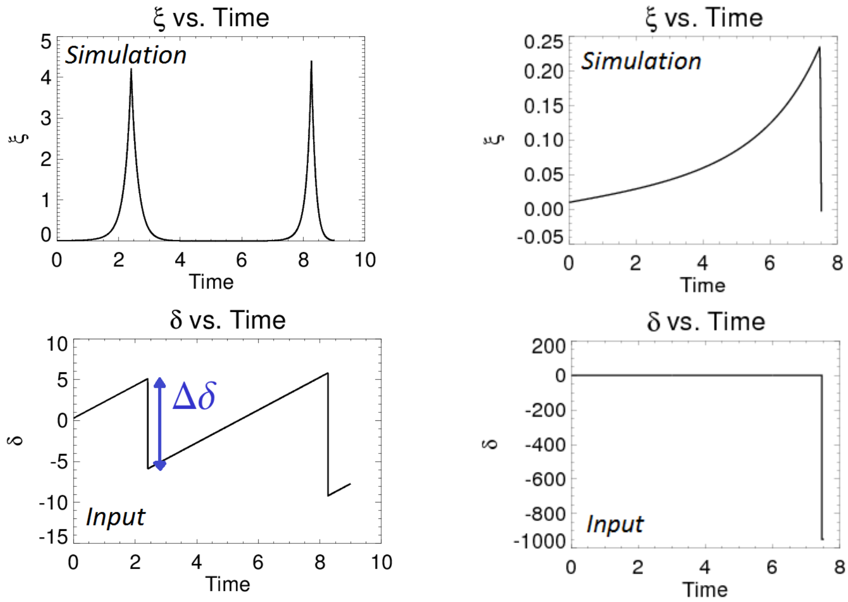
ELM Cycle Model

Here, we propose a semi-heuristic model to simulate an ELM cycle, neglecting the small scale viscosity in the system. Starting from marginal stability, we increase the pressure gradient linearly in time: $\delta(t) = s \cdot t$, until the quadratic non-linearity term and the linear term balance. At this time we assume that the eruption starts with an associated enhanced transport. We do not address the transport mechanism, but impose a pressure gradient crash until the instantaneous growth rate as predicted by Eq. (1) is zero: $\delta = s \cdot t - \Delta\delta$. After this drop, the system is linearly stable, and the pressure gradient builds towards the next crash and the cycle repeats (Fig. 4a/ 4b). The width $\Delta\psi$ and the drop in pressure gradient $\Delta\delta$ provide a prediction for the energy ejected in ELM.

4. Conclusion

We have demonstrated that the viscosity influences the evolution of the filament amplitude but also the widths $\Delta\alpha$ and $\Delta\psi$. However this change of the evolution differs from theoretical prediction based on balancing terms in the nonlinear evolution equation.

Additionally, we have introduced a semi-heuristic model which enables us to simulate a full ELM cycle. No knowledge of the transport mechanism responsible for the crash is required for this model. Nevertheless, the model provides a prediction for the drop in the pressure gradient which allows us to estimate



(a) *Two eruptions of the displacement vector and the pressure gradient evolution; weak shaping*

(b) *One eruption and the drop in the pressure gradient; strong shaping-oscillates after the crash in this case*

Figure 4: Predictions from our heuristic model of the ELM cycle.

the energy which is released during an ELM.

The next step is to develop codes to calculate the coefficients of the ballooning mode envelope equation for realistic tokamak geometry. These coefficients are field line averaged quantities which include most of the geometry of the fusion devises. We can then make predictions for ELM sizes in experiments like MAST and ASDEX Upgrade. If successfully benchmarked against the experimental data, this method would provide predictions for ELMs in ITER.

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