Impact of the plasma $\beta$ and magnetic topology on the nature of plasma edge turbulence

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Experimental measurements of fluctuation levels on typical fusion devices reveal that magnetic perturbations are typically much smaller than electrostatic perturbations. However, as even small magnetic fluctuations ($\approx 10^{-4}$) can locally modify the topology of the magnetic surfaces, they can thus play an important role with respect to the transport properties of the plasma. The nature of turbulence at plasma edge is not totally clear, even if drift-wave turbulence is assumed to be dominant [2]. The resistive ballooning instability is a possible driving mechanism of the turbulence in the plasma edge region [1]. Indeed, recent works related to the characterization of the L-H transition revealed that resistive ballooning modes are plausible candidates for the origin of edge turbulence [3]. Key ingredients for the destabilization of resistive ballooning modes appear to be the values of the safety parameter and the collisionality at the edge[3].

In the present work, the nature of edge turbulence is investigated through numerical simulations in toroidal geometry using realistic plasma edge parameters in high collisionality regimes. These simulations focus on the effects of plasma $\beta$ parameter and magnetic topology on the nature of plasma edge turbulence. Presented results have been performed with EMEDGE3D [4], a three dimensional global code which calculates the evolution of the pressure, the electrostatic potential and self-consistent electromagnetic fluctuations at plasma edge.

Electromagnetic plasma edge turbulence is simulated here using a resistive ballooning model. The latter is based on fluid equations for the normalized electrostatic potential $\phi$, electromagnetic flux $\psi$, electronic pressure $p_e$ and parallel velocity $v_\parallel$. [4].

\begin{align}
\frac{d}{dt} W + \{\phi, W\} &= -\nabla_\parallel J - \omega_\parallel \mathbf{G} p + \nu \nabla_\perp^2 W, \\
\partial_t v_\parallel + \{\phi, v_\parallel\} &= -(1 + \varepsilon_T) \nabla_\parallel p + \nu \nabla_\perp^2 v_\parallel, \\
\partial_t \psi &= -\hat{\beta}^{-1}_s \nabla_\parallel (\phi - p_e) + \eta J, \\
\partial_t p_e + \{\phi, p_e\} &= -\Gamma \nabla_\parallel (J + v_\parallel) + \Gamma \omega_\parallel \mathbf{G} (\phi - p_e) + \chi_\perp \nabla_\perp^2 p_e + S(r),
\end{align}
Equations (1 and 2) correspond to the normalized charge balance, Eq. (4) is the normalized energy balance and Eq. (3) corresponds to the Ohm’s Law. $\nabla_\perp$ and $\nabla_\parallel$ respectively correspond to the parallel and perpendicular gradients along field lines. $G$ is the curvature operator, $\nu$ represents the viscosity, and $\chi_\perp$ the perpendicular thermal diffusivities. Time is normalized by $\tau = L_\perp/c_S$, where $c_S$ is the sound speed, and $L_\perp$ is the pressure gradient length. The perpendicular and parallel length scales are $\rho_s$ and the magnetic shear length $L_s$, respectively. Here, $\eta$ is the parallel resistivity. $q(r)$ stands for the safety factor which measures the magnetic field line pitch. $\hat{\beta}_s$ is expressed by $\hat{\beta}_s = \beta \frac{L_s^2}{2L_\perp}$ where $\beta$ is the classical $\beta$ parameter (the ratio of kinetic pressure to the magnetic pressure). The parameter $\delta_c$ is defined by $\delta_c = 2\Gamma L_\perp/R_0$, $\Gamma$ is the heat capacity ratio ($\Gamma = 5/3$) and $\omega_D = 2L_\perp/R_0$. The coefficient $\epsilon_T$ represents the ratio between assumed electronic and ionic temperatures (here, $\epsilon_T = 1$). The last term of the r.h.s. of Eq. (3), $S(r)$ represents a constant energy source.

The two relations for vorticity and current are given by,

$$ W = \nabla_\perp^2 \phi \quad , J = \nabla_\perp^2 \psi $$

where $d_t = \partial_t + \mathbf{u}_E \cdot \nabla = \partial_t + \{ \phi, \cdot \} , \mathbf{u}_E \cdot \nabla$ representing the advection by $\mathbf{E} \times \mathbf{B}$ drift.

Magnetic flux surfaces are modeled by a set of concentric circular torii, where the coordinates $(r, \theta, \varphi)$ correspond to the minor radius, the poloidal and toroidal angles respectively. In that case, the normalized operators are

$$ \nabla_\parallel = \partial_\varphi + \frac{1}{q(r)} \partial_\theta \{ \psi, \cdot \} \quad , \quad \nabla_\perp^2 = \partial_r^2 + 1/r^2 \partial_\theta^2 $$

$$ G = \sin \theta \partial_r + 1/r \cos \theta \partial_\theta \quad , \quad \{ f, g \} = 1/r (\partial_r f \partial_\theta g - \partial_\theta f \partial_r g) $$

The safety factor $q(r)$ is assumed to increase monotonically in a domain between $q = 2.5$ and $q = 6$ at the plasma edge. However in the presented work, the influence of the mathematical function used to represent this factor is simulated through the comparison between two $q$ profiles.
\[ q_1(r) = \frac{1}{(ar + b)}, \quad q_2(r) = ar^2 + br + c \]

In both cases, constants \( a, b \) and \( c \) are choosen such as \( q(r/a) = 2.5 \) at \( r/a = 0.47 \) and \( q(r/a) = 4 \) at \( r/a = 0.92 \). The main difference between them will be the associated current profiles.

As mentionned in the first part of this paper, the values of the safety parameter and collisionality at the edge appear to be key ingredients for the destabilization of resistive ballooning modes. For that purpose, the normalized coefficient used are derived from experimental measurement made on Tore Supra machine (shot TS#36086). The assumed resistivity is of the order of \( \eta_0 \approx 1.5e^{-7} \), the electronic temperature at the edge \( T_e \approx 310keV \), a magnetic field \( B_0 \) of \( B_0 = 3T \) and \( \beta \approx 6.2e^{-4} \). Results of linear simulations are presented here in the aim to characterize the numerical results. Energy spectrum are presented in Fig. (3) and a map of the averaged phase shift distribution between pressure and electrostatic potential in function of the poloidal mode are presented in Fig. (4).

As first indication, the representation of modes amplitudes as function of poloidal \( (m) \) and toroidal \( (n) \) mode number reveals that the unstable modes are resonant ones satisfying \( 2n < m < 4n \). Moreover, probability distribution of phaseshift between electronic pressure and electrostatic potential close to \( \pi/2 \) is in agreement with a resistive ballooning instability.

The linear growth rates are plotted on Fig. (2). Fig. (2a) correspond to the case where the safety factor is given by \( q(r/a) = q_1(r/a) \). In that case, it appears that there is like a bifurcation in the linear growth rate when \( \beta_e \) increases. This bifur-
cation is not observed in the case presented on Fig. (2b) where $q(r/a) = q_2(r/a)$. This can be explained by a critical value of $\beta$, which depends on magnetic topology.

In conclusion, turbulence simulations of a tokamak edge plasma have been realized focusing on the impact of magnetic topology and electromagnetic fluctuations on the nature of observed turbulence. These simulations have been focused in the aim to verify the possible generation of resistive ballooning instability at plasma edge.

These preliminary results are in agreement with the possibility of resistive ballooning modes as a drive for plasma edge turbulence. The impact of the $\beta$ value, coupled with the $q$-profile used reveals a bifurcation in the instability source. A possible explanation, is a transition from a transport dominated by Reynolds stress and electrostatic components to a transport dominated by Maxwell tensor term and ideal magnetohydrodynamics effects. The next studies will focused on a better characterization of the bifurcation with a better characterization of measured transport and eigen modes observed in both cases.

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