Effect of Density Fluctuations in the Scrape-Off Layer on the Lower Hybrid Power Spectrum

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Introduction
The Lower Hybrid (LH) wave is launched at radio frequencies, 1 – 5 GHz, to drive a toroidal current in tokamak plasmas. Experimentally, the LH wave has been successfully used to extend the lifetime of the plasma in particular in the superconducting tokamak Tore Supra (TS) [1, 2]. However, the interpretation of the level of the LH driven current, as well as its density profile, remains still challenging in existing machines, especially for dense plasmas which are usually characterized by a low electron temperature \( T_e \). Consequently, reliable interpretations of reactor grade scenarios in existing tokamaks are particularly difficult.

The power spectrum of the LH wave is launched at low values of the parallel refractive index, \( n_\parallel \approx 2 \), to achieve high CD efficiency. A tail of fast electrons is pulled from the thermal bulk via the usual Landau damping mechanism. However, the wave is only strongly absorbed at the refractive index \( n_\parallel \approx 5.5/\sqrt{T_e [keV]} \), [3], thus resulting in the so-called LH spectral gap \( \delta n_\parallel \approx n_\parallel - n_\parallel \). Toroidal refraction has shown to be the most effective mechanism that fills the spectral gap, though never unambiguously demonstrated. Inherent in the ray tracing propagation code coupled to a Fokker Plank solver of the resonant kinetic absorption, toroidal upshift bridges only moderate spectral gap width as shown by the quantitative assessment of the aforementioned codes versus experimental observations of various moments of the suprathermal tail [4, 5].

However, such a good agreement fails when the spectral gap becomes large, like in high density plasmas, because of the concomitant low \( T_e \). In this case, the stochastic nature of the ray trajectory develops well before the full absorption of the LH wave, leading to almost unpredictable simulation results, in full contradiction with the experimental phenomenology which exhibits smooth and progressive parametric dependencies with both plasma parameters or wave launching conditions. In this regime, the rays perform multiple bounces between caustics and cut-off leading to a rapid loss of the wavefront topological structure, while the cumulative numerical errors may become large. \( n_\parallel \) upshift due to toroidal refraction is therefore too weak to bridge the spectral gap, and other physical effects most likely predominate [5, 6].

A particularly enlightening case concerns the TRIAM-1M tokamak, whose very large aspect ratio prevents any significant toroidal \( n_\parallel \) upshift, while full LHCD experiments have been routinely achieved with a very large \( \delta n_\parallel \). Both full-wave or ray racing calculations are unable to explain the absorption of the LH wave, except if \( \delta n_\parallel \) is supposed already small at the separatrix [7]. Since the LH dynamics in TS tokamak for high density regimes have some similarities with the TRIAM-1M ones, the impact of a spectral gap already bridged in the scrape-off layer (SOL) has been quantitatively studied, taking benefit of the large experimental database, in particular the fast electron bremsstrahlung tomography [5]. A very good agreement between simulations and observations is found when the tail part of the LH power spectrum exceeds 50% of the total launched power, which suggests that the narrow initial spectrum excited by the antenna is likely significantly modified by some physical process which takes place in the SOL in front of the antenna. A detailed analysis has shown that broadening of the power spectrum at the separatrix for ray tracing calculations may be interpreted as the consequence of a fast fluctuating LH power spectrum with respect to the electron slowing down time in the plasma [5].

In this context, the possible impact of local electron density fluctuations along the magnetic field lines in the SOL on the LH wave spectral properties at the plasma separatrix is investigated quantitatively, aiming at validating the heuristic approach used for TS LHCD simulations [5].
Scrape-off plasma model and code validation

Modeling the propagation of the LH power spectrum in the scrape-off layer in front of the antenna is performed with the finite element solver COMSOL Multiphysics®, [19]. The full wave equation is solved for a $2-D$ slab model with a cold plasma dielectric tensor, where the toroidal direction $\hat{z}$ is assumed to be parallel to magnetic field lines (safety factor $q \gg 1$), the poloidal direction $\hat{y}$ is infinite, while the radial one is along $\hat{x}$. In order to mimic an infinite medium within the finite integration box and avoid spurious wave reflections at the boundaries, an outermost layer is added where all LH power is fully absorbed.

The model is validated against ALOHA 1–D coupling code, [20], with a quiescent plasma for both FAM (fully active multi-junction) and PAM (passive active multi-junction) like antennas. Based on TS parameters, LH frequency is 3.7GHz, density is linearly increasing in the radial direction with a decay length $\lambda = 20\text{mm}$, and the reference density, $n_{e0}$ is $1.2n_c$ for FAM and $n_c$ for PAM, where $n_c$ is the cut-off density. A fast Fourier transform is applied to power values obtained with COMSOL, and collected along lines parallel to the $\hat{z}$ direction, in order to calculate the power spectra $dP/dn_z$. Figure (Fig. 1), shows full agreement between the power spectra calculated by COMSOL and those with ALOHA. The comparison is done for three module phasing values with $0, \pm \pi/6$ difference from $\phi_{mod,0}$, which corresponds to highest directivity ($\phi_{mod,0} = 3\pi/2$ for FAM and $\pi$ for PAM). Note that $n_{e0}$ takes the values 1.92, 2.01, 2.11 for FAM and 1.63, 1.72, 1.81 for PAM. The agreement is also excellent for the negative side of $dP/dn_z$.

The parallel density fluctuation model

Up to now, the effect of electron density or magnetic fluctuations on the LH wave dynamics has been investigated with a fluctuation model characterized by $k_{||} \simeq 0$ corresponding to electron drift wave perturbations [6]. This can lead only to a rotation of $k_{\perp}$ with a possible mode conversion, but the impact on the spectral gap is only indirect, by a cumulative effect along the ray trajectory, taking benefit of the intrinsic stochasticity of the wave dynamics in the toroidal geometry. Quantitative calculations have shown that the overall effect of such fluctuations is too weak to remove the sensitivity to initial conditions, and the predicted current density profile still disagree with observations [6]. Therefore, only a density perturbation along the magnetic field line could impact directly the LH power spectrum, and contribute to bridge rapidly the spectral gap. So far, due to the fast transport along the magnetic field lines, the electron density is generally supposed to be homogeneous along the magnetic field direction, or any gradient is considered to have a characteristic scale length much longer than the LH wavelength. However, this statement is not necessarily valid in the SOL, where the interaction of the plasma with the surrounding wall may deeply change the standard picture. Few experimental details are available concerning the characteristics of the fluctuations in the SOL. In general, existing diagnostics
have allowed to investigate perturbations in the poloidal or radial directions, showing propagation of self-consistent filamentary outward density bursts called blobs, near and far SOL [8, 9, 10]. In general, blobs are almost aligned with magnetic field lines, however observations in TORPEX machine have revealed that in a resistive interchange mode regime, $\tilde{k}_\parallel \neq 0$ could exist [11, 12]. Furthermore, [13] states that the blob vorticity may change size as it propagates, in particular in presence of density gradient along the blob propagation path, [10], while several analytical studies suggest an impact of turbulence on parallel fluctuations, [14, 15, 16, 17]. On the other hand, the decay length of the density in the SOL is strongly perturbed in front of the LH antenna, which suggests a local perturbation in the toroidal direction could take place [18]. Electron density fluctuations in the SOL are incorporated in front a FAM like antenna. Due to high calculation effort, a flat background density and two antenna modules are considered (larger initial spectral width as compared to the actual configuration), as shown in figure (Fig. 2). The background density is set to $n_{e0} = 2.6n_e$ to avoid reflection due to densities close to cut-off at high fluctuation amplitude (up to 50%), while the module phasing is $\phi_{\text{mod,0}}$, and the SOL depth is $8 \text{ cm}$ in consistence with TS parameters. Fluctuations are modeled as a thin perturbed layer along $\xi$, assuming monochromatic wave-like parallel dependence as shown in figure (Fig. 3). The total density is the sum $n_e = n_{e0} + \bar{n}_e$, where

$$\bar{n}_e = \sigma_n n_{e0} \sin(\tilde{k}_z z + \tilde{\phi}) \exp\left(-\frac{(x - \bar{x})^2}{\Delta \tilde{x}}\right),$$  

while $\tilde{k}_z$ is the parallel wave number, $\sigma_n \in ]0,1[$ is the perturbation amplitude and $\tilde{\phi} \in ]0,2\pi[$ the phase. A Gaussian radial dependence is assumed, where $\bar{x}$ and $\Delta \tilde{x}$ are the radial position and FWHM of the perturbation respectively.

**Effect of fluctuations on the LH power spectrum**

The effect of a fluctuating layer on the LH power spectrum is first studied with a simple monochromatic perturbation characterized by the following parameters: $\tilde{k}_z = 0.5k_0$, $\sigma_n = 0.5$, and $\bar{x} = 0.375$, $\Delta \tilde{x} = 0.125$. All lengths are normalized to the SOL depth, and $k_0 = 2\pi / \lambda_0$, where $\lambda_0$ is LH wavelength in vacuum ($\sim 8 \text{ cm}$). In figure (Fig. 2) the density perturbation is shown, while (Fig. 3) shows the resulting electric field amplitude for a random phase. Diffraction effects are clearly visible, which result in a strong modification of the power spectrum as the LH wave propagates to the separatrix.

In a quiescent plasma, the LH power spectrum, which is peaked at $n_{e0} \approx 2.0$ with negligible amount of power in the tail, remains unchanged from the antenna to the separatrix. Conversely, a significant fraction of the LH power in the main peak is transferred to multiple satellite peaks in $dP/dn_e$ once the wave propagates through the perturbed region, an irreversible process until the separatrix is reached. As shown in figure (Fig. 4), the LH power spectrum exhibits large fluctuations depending upon the perturbation phase, a result which is fully consistent with the heuristic quantitative analysis of LHCD TS discharge [5]. Interestingly, the density fluctuations in the

![Figure 4: (a), (b), (c), (d) : Instantaneous power spectra for different phases $\tilde{\phi} / \pi = 0.28, 1.9, 0.97, 1.5$ and at different positions in the SOL.](image)
SOL have no effect on the power spectrum in front of the antenna, and the coefficients of the scattering matrix must remain consistent with the theory based on a quiescent local plasma, as observed experimentally.

In order to evaluate the evolution of the phase averaged LH power spectrum with the spectral properties of the fluctuations, a scan in $\tilde{k}_z$ has been performed, summing over twenty random phases uniformly distributed between 0 and $2\pi$.

First, a Gaussian fit of distinct lobes in the phase averaged power spectrum is done, and the corresponding mean and standard deviation are deduced for each lobe. Consequently, the mean and standard deviation of the tail denoted $\mu(n_{||})$ and $\sigma(n_{||})$ respectively, are those of the Gaussian equivalent of the lobes constituting it. The tail evolution is shown in figure (Fig. 5) where a large fraction of the total power, of more than 50%, is transferred to the tail for $\tilde{k}_z < 0.5k_z0$, while for $\tilde{k}_z < 0.2k_z0$, most of the tail power is due to broadening of the main lobe. As $\tilde{k}_z$ increases, the tail satellite peaks gradually move to higher $n_{||}$ which leads to shifting $\mu(n_{||})$ upwards, and to changing $\sigma(n_{||})$, which peaks at $\tilde{k}_z \approx 0.4k_z0$, indicating significant broadening in the tail.

In conclusion, a $2-D$ slab model of the SOL and a FAM like antenna is modeled in COSOL Multiphysics®, in presence of a thin perturbed electron density layer. When the Fourier component of the parallel density fluctuation is of the same order of the LH wavelength, the power spectrum is strongly modified by diffraction. As a result, the LH power spectrum at the separatrix exhibits satellite peaks carrying a significant amount of power, as inferred from ray tracing and Fokker-Planck calculations [5]. The perturbation of the density along the magnetic field lines in the SOL is therefore a plausible mechanism to bridge the spectral gap at the plasma edge, before the wave propagates inside the separatrix. Further experimental assessments are necessary to validate this analysis.

References