Quasi-linear analysis of the extraordinary electromagnetic wave destabilized by runaway electrons

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Introduction Relativistic runaway electron distributions are strongly anisotropic in velocity space. Due to this anisotropy they can give rise to electromagnetic waves via a resonant interaction, which in turn can cause the velocity-space scattering of runaway electrons. Previous studies considered the destabilization and evolution of the whistler wave (whistler wave instability, WWI), which indeed leads to rapid pitch-angle scattering of the runaways [1]. However, in a recent paper it was shown that runaways can destabilize the so-called extraordinary electron (EXEL) wave with several orders of magnitude lower runaway density than needed for the WWI in a low electric field case [2]. It is interesting to analyze what effect the EXEL wave could have on runaway beam formation in the presence of a high electric field, typical of tokamak disruptions.

Growth rate The waves investigated are described by their wave dispersion in a homogeneous, magnetized, cold plasma approximation [3]:

$$\left[\left(\epsilon_{11}-\frac{k_\parallel^2c^2}{\omega^2}\right)\left(\epsilon_{22}-\frac{k^2c^2}{\omega^2}\right)+\epsilon_{12}^2\right]\left(\epsilon_{33}-\frac{k_\perp^2c^2}{\omega^2}\right)-\frac{k_\parallel^2k_\perp^2c^4}{\omega^4}\left(\epsilon_{22}-\frac{k^2c^2}{\omega^2}\right)=0, \qquad (1)$$

where the frequently used electromagnetic approximation, $\epsilon_{33}\gg n^2\cos\theta\sin\theta$ was relaxed. Here k is the wave number, $k_\parallel$ denotes its part parallel to the static magnetic field, ω is the wave frequency, c is the speed of light, ϵ_{ij} are the elements of the dielectric tensor.

As it was previously shown in Ref. [2], one of the branches of the above dispersion relation, the extraordinary electron (EXEL) wave, is prone to destabilization by the anisotropic runaway electron distribution much easier than the whistler wave. The EXEL wave propagates with frequencies $\omega\simeq\omega_{pe}$ for low magnetic fields, but its dispersion changes slightly for high magnetic fields (Fig. 1ab). Figure 1c shows the linear growth rate of the EXEL wave for an avalanching runaway distribution, valid for high electric fields, taken from Ref. [4].

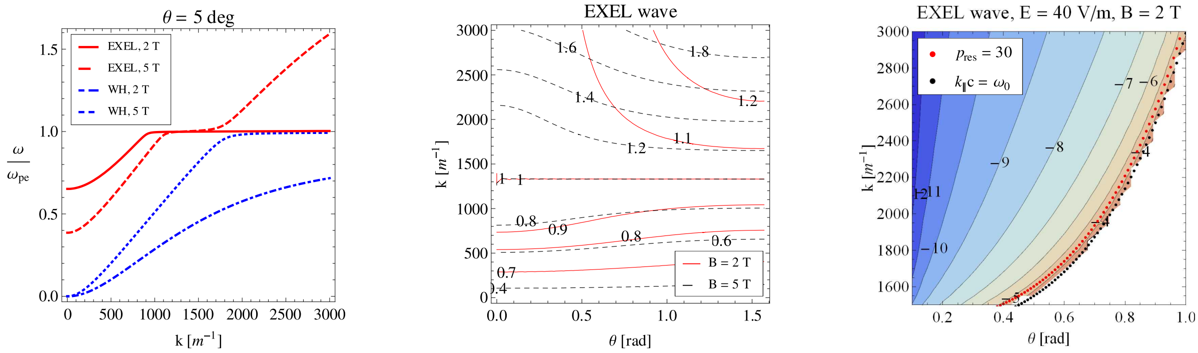

Figure 1: *(a) Dispersion of the two lowest frequency branches (electron-whistler [5] and EXEL wave) for two magnetic field values, (b) Normalized dispersion of the EXEL wave (ω/ω_{pe}) for different magnetic fields, (c) Growth rate of the EXEL wave ($\ln[\gamma_i/\omega_{ce}]$). The parameters are thermal electron density $n_e = 5\cdot 10^{19}\,\mathrm{m}^{-3}$, effective charge $Z = 1$, runaway density $n_r = 3\cdot 10^{17}\,\mathrm{m}^{-3}$, Coulomb logaritm $\ln\Lambda = 18$ and normalized maximum runaway momentum $p_{\max} = 30$.*

The wave number space for the EXEL wave can be divided into two parts: the low wave number region where $\omega_0 > k_\parallel c$ and the high wave number region where $\omega_0 < k_\parallel c$. By calculating the growth rate in both regions we find that it is orders of magnitude higher in the latter region. As the $k_\parallel c = \omega_0$ line is approached the growth rate increases. At the same time, the momentum of the runaways needed for the destabilization of the wave (resonant momentum) becomes higher. Thus, the most unstable EXEL wave is destabilized by the maximum energy runaways. For a runaway electron distribution with maximum runaway energy 15 MeV corresponding to $p_{\max} = 30$, the parameters of the most unstable wave are wave-number $k \sim 3000\,\mathrm{m}^{-1}$, propagation angle $\theta \sim 1.0$ and frequency $\omega_0 \sim 5\cdot 10^{11}\,\mathrm{s}^{-1}$.

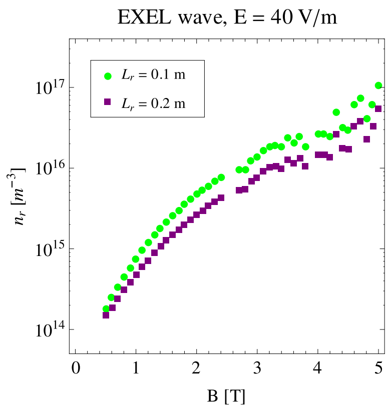

Figure 2: *Stability thresholds for the EXEL wave in high electric field for different runaway beam radii (L_r) for $T_e =$ 20 eV temperature, $n_e = 5\cdot 10^{19}\,\mathrm{m}^{-3}$, $Z = 1$, $p_{\max} = 30$.*

Comparing the growth rate of the EXEL wave to collisional [6] and convective damping [1] yields a critical runaway density needed for the destabilization of the most unstable wave. Figure 2 gives the critical densities for the EXEL wave for the avalanching distribution from Ref. [4]. In agreement with previous results in the low electric field case [2], the number of runaway electrons needed for destabilization is significantly lower for the EXEL wave also in the high electric field case considered in this paper.

Quasi-linear analysis In the ultrarelativistic limit $\gamma \simeq |p_\parallel|$, the effect of the EXEL wave on the runaway distribution is given by the following diffusion equation [1,3]: $\frac{\partial f}{\partial \tau} = \frac{1}{p_\perp}\frac{\partial}{\partial p_\perp}\left(p_\perp \frac{\partial f}{\partial p_\perp}\right)$. Here $\tau(p_\parallel) = \int_0^t dt' \tilde{D}(p_\parallel, t')$ and $\tilde{D}(p_\parallel, t')$ is a quasi-linear diffusion coefficient [1]. $\tilde{D}$ depends on the wave energy and polarization, and is calculated along the $k_{\rm res}(\theta)$ resonant curve, which is the solution of $(\omega_0 + \omega_{ce}/p_\parallel - k_\parallel c) = 0$ for a fix $p_\parallel$ value. The main difference from the analysis in Ref. [1] is the polarization of the wave, which for the EXEL wave is $(1, E_y/E_x, E_z/E_x) = \left(1, -i\frac{\omega_{pe}^2\omega_{ce}/\omega_0}{\omega_0^2 - k^2c^2 - \omega_{ce}^2 - \omega_{pe}^2 + k^2c^2\omega_{ce}^2/\omega_0^2}, \frac{k_\parallel k_\perp c^2}{\omega_{pe}^2 + k_\perp^2 c^2 - \omega_0^2}\right)$.

The numerical scheme solving the above equations is as follows: For each $p_\parallel$ value in each time step the linear growth rate of the EXEL wave is calculated along the $k_{\rm res}(\theta)$ resonant curve. The wave energy is determined as the solution of $\partial W_k/\partial t = 2\gamma_l(t)W_k$, with the initial condition $W_k(t=0) = k_B T_e/2$, which is the energy level of thermal fluctuations. The wave energy is then integrated along the resonant curve together with the polarization vector, resulting in the diffusion coefficient. The time integral of this diffusion coefficient is the dimensionless time, $\tau(p_\parallel)$. Solving the diffusion equation, the time evolution of the runaway distribution can be given as a function of $\tau(p_\parallel)$.

Figure 3 shows the time evolution of the runaway distribution. The destabilization of the EXEL wave results in the momentum-space scattering of runaways to higher perpendicular momentum. This effect starts around the maximum runaway momentum ($p_{\rm max} = 30$), then moves to lower values. After the wave becomes stable (its growth rate minus the damping rates turns negative), the wave energy starts to decrease and it has no effect on the distribution until the second destabilization of the wave. When the runaway distribution once again grows to high enough amplitude to destabilize the wave, it will have a second scattering effect on the runaways. This means that there is a periodic effect on the runaway distribution: each destabilization of the EXEL wave causes the momentum-space scattering of the runaways, followed by a phase when the wave is stable and the distribution can grow until another destabilization of the wave. Note that the time-scale of this resonant interaction is very short.

Conclusions The high-frequency EXEL wave can be destabilized by significantly less runaways than the whistler wave previously investigated in Ref. [1,4]. This means that in a tokamak disruption it should be destabilized well before the whistler wave. When the EXEL wave grows to high enough energy, it causes the momentum-space scattering of the runaway electrons. If the resonant interaction is sufficiently strong, it may even limit the number of runaway electrons. It may be possible to detect the phase-space scattering of the runaways by analyzing their synchrotron spectrum, as high momentum runaways have significant effect on it. As the critical density for this interaction is lower for low magnetic fields, signs of the phase-space scattering should be easier to find for low magnetic fields. Detection of these waves, or signs of the resonant interaction would give indications about the possibility of stopping the runaway beam formation via waves in ITER.

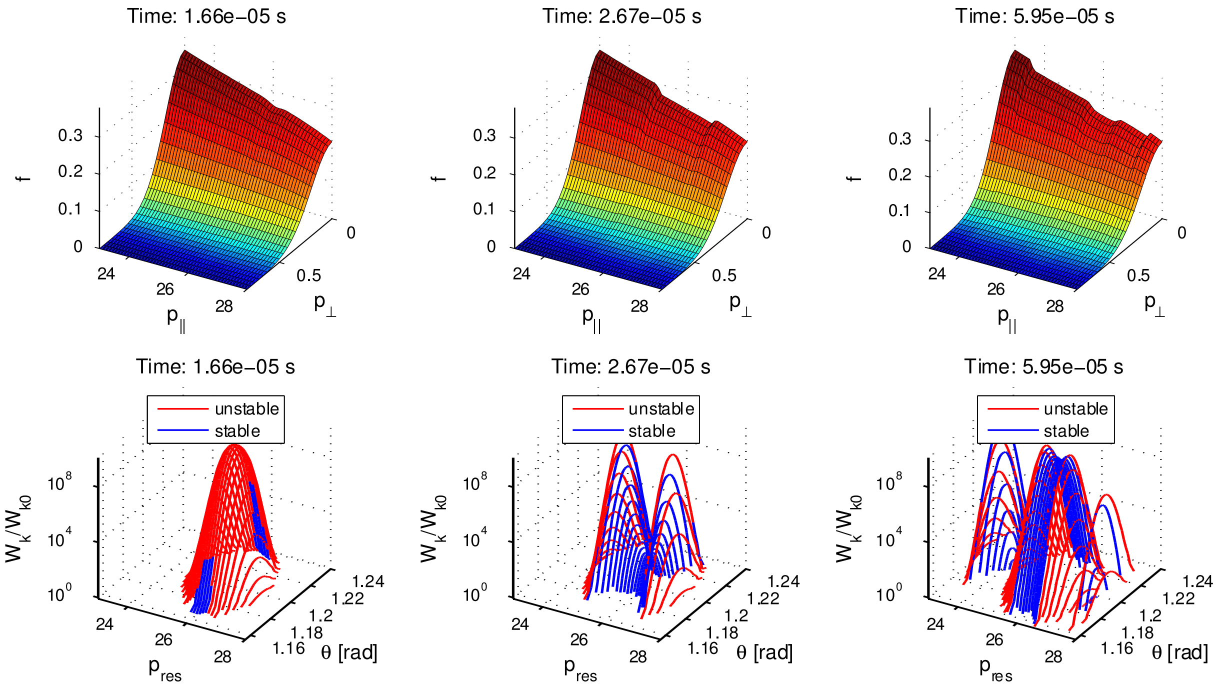

Figure 3: *Quasi-linear evolution of the runaway distribution and the wave energy of the EXEL wave. Red and blue lines correspond to the wave energy along the* k_{res} *resonant curve as a function of* θ *for a fix* p_{res}*. The runaway momentum, p is normalized with* $m_e c$*. The displayed time corresponds to the time elapsed since the first destabilization of the most unstable wave. The parameters are* $B = 2\,\mathrm{T}$, $n_e = 5\cdot 10^{19}\ m^{-3}$, $Z = 1$, $T_e = 20\,\mathrm{eV}$ *and* $p_{\max} = 30$.

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