GOLDEN AND NOBLE MAGNETIC SURFACES AND MAGNETIC SHEAR IN TOKAMAKS

Halima Ali\textsuperscript{1} and Alkesh Punjabi\textsuperscript{1}

\textsuperscript{1}Hampton University, Hampton, VA, 23668, USA

ABSTRACT: A new symplectic map in magnetic coordinates is developed to study the resilience of golden and noble magnetic surfaces in tokamaks against resonant magnetic perturbations. The parameter $s$ in this logarithmic map controls the magnetic shear. The locations of the equilibrium magnetic surfaces with golden and noble values of safety factor $q$ [F.A. Volpe, J. Kessler, H. Ali, T.E. Evans and A. Punjabi, \textit{Nucl. Fus.} \textbf{52}, 054017 (2012), H. Ali and A. Punjabi, to appear in REDS: Inc. Plasma Sc. Plasma Tech] are calculated for varying magnetic shear. Magnetic perturbations with resonant surfaces just above and below these golden and noble surfaces are considered. These perturbations are applied with increasing amplitude and the critical levels of the perturbations $\delta_{\text{crit}}$ when the surfaces break down are found. As the magnetic shear is increased, the resonant surfaces come closer. There is competition between the closer proximity of resonant surfaces and the magnetic shear. The results of this study will be presented.

Magnetic shear plays an important role in magnetic confinement of fusion plasmas. In this paper, we construct a symplectic map in magnetic coordinates that calculates magnetic field line trajectories in single-null divertor tokamaks with variable shear. We use this map to study the resilience of golden magnetic surface in tokamaks against resonant magnetic perturbations. The mapping technique [1] is applied to integrate the field lines of toroidally confined plasma. In this formalism the equations for magnetic field lines take the Hamiltonian form

$$\frac{d\psi}{d\phi} = -\frac{\partial \chi}{\partial \theta}, \quad \frac{d\theta}{d\phi} = \frac{\partial \chi}{\partial \psi},$$

(1)

where $\psi=r^2/2$ is a toroidal magnetic flux canonically conjugated to the poloidal angle $\theta$ and $\phi$ is the toroidal angle. The field lines Hamiltonian

$$\chi = \chi_0(\psi) + \chi_1(\psi, \theta, \phi)$$

(2)

can be represented as a sum of the unperturbed flux

$$\chi_0 = \int \frac{d\psi}{q(\psi)}$$

(3)

and the perturbed part of the flux
\[
\chi_1(\psi, \theta, \phi) = \sum_{m,n} \chi_{mn}(\psi) \cos(m \theta - n \phi + \xi_{mn}),
\]

Where \( \chi_{mn}(\psi) \) is the amplitude of the perturbation with mode numbers \((m,n)\) with the phase \( \xi_{mn} \).

The map for the field line is in magnetic coordinates, and is given by
\[
\psi_{n+1} = \psi_n - k \frac{\partial \chi(\psi_{n+1}, \theta_n, \phi_n)}{\partial \theta_n}, \quad \theta_{n+1} = \theta_n + k \frac{\partial \chi(\psi_{n+1}, \theta_n, \phi_n)}{\partial \psi_{n+1}},
\]

where \( k \) is the map parameter. It represents the step-size of integration. Here, \( k \) is kept fixed at \( k = 2\pi/36 \).

In this paper, the safety factor \( q \) as a function of magnetic coordinate \( \psi \) is given by
\[
q(\psi) = 1 - s \ln(1 - \psi)
\]

The parameter \( s \) in this logarithmic map controls the magnetic shear and \( \psi \) is normalized by \( \psi_{SEP} \), the toroidal flux inside the separatrix surface. The safety factor at the magnetic axis, \( q(\psi=0) \), is equal to unity. Fig. 1 shows safety factor as function of \( \psi \). The equilibrium generating function is then given by
\[
\chi_{EQ}(\psi) = \frac{1}{s} e^{-1/s} \left\{ \text{Ei} \left[ \ln(1 - \psi) - \frac{1}{s} \right] - \text{Ei} \left[ -\frac{1}{s} \right] \right\}. \tag{6}
\]

The exponential integral \( \text{Ei}(x) \) in eqn.(6) is defined as \( \text{Ei}(x) = \mathcal{P} \int_{-\infty}^{x} \frac{e^{-t}}{t} dt = \mathcal{P} \int_{x}^{\infty} \frac{e^{t}}{t} dt \). This definition can be used for real nonzero values of the argument \( x \), but the integral has to be understood in terms of the Cauchy principal value due to the singularity of the integrand at zero. Fig. 1(b) shows the equilibrium generating function \( \chi_{EQ} \) as a function of \( \psi \). For magnetic perturbation, we choose two tearing modes with mode numbers \((m,n)\)={\(3,2\),(2,1)\)}, and
\[
\chi_1(\theta, \phi) = \delta \left[ \cos(3\theta - 2\phi) + \cos(2\theta - \phi) \right]. \tag{7}
\]

Typically, for standard H-modes, \( q \) on the magnetic axis is the minimum \( q \)-value and is just below unity, while \( q \) close to the plasma boundary, \( q_{95s} \), is 3 or above. \( q_{95} \) is the safety factor of the surface that has 95% of the poloidal flux compared to the flux inside the separatrix. Magnetic shear and \( q_{95} \) play an important role in plasma stability and confinement. Experiments are beginning to access the external kink stability boundary at edge safety factor \( q_{95}=5 \) and in [2], authors reported results of experiments performed at \( q_{95} \approx 9 \) in JT-60U, in order to attain high enough \( \beta_p \), which is significantly higher than that observed in the baseline scenario for diverted
machines or in the ITER design. Fig. 2 shows safety factor at the 95% flux surface, $q_{95}$, as a function of the shear parameter $s$ for the generating function given in eqn. (6) for our map.

Fig. 2. The $q_{95}$ as a function of toroidal flux for the shear parameter $s$. Curve through the data points is a power fit: $q_{95}$ scales as $s^{2/3}$.

From Fig. 2, we see that for $s=0.7$, $q_{95}=2.58$ and for $s=3.5$, $q_{95}=7.1$. The locations of the resonant magnetic surfaces are calculated for varying magnetic shear. The locations of the equilibrium magnetic surface with golden values of safety factor $q$ [3,4] are calculated for varying magnetic shear. We call these surfaces golden surfaces. As the magnetic shear parameter is increased from 0.7 to 3.5, the resonant surfaces come closer. There is competition between the closer proximity of resonant surfaces and the magnetic shear. See Fig. 3. The perturbation given by eqn.(7) is applied to these surfaces with increasing amplitude; and the critical levels of the perturbations $\delta_{\text{crit}}$ when the surfaces break down are calculated. We found that as the shear
parameter increases, the critical perturbation, $\delta_{\text{crit}}$ scales approximately exponentially with the shear parameter, $s$. See Fig. 4.

![Graph showing resonant and equilibrium golden magnetic surfaces as a function of the shear parameter $s$.](image1)

![Graph showing distance between resonant magnetic surfaces as a function of the shear parameter $s$.](image2)

![Graph showing the critical amplitude $\delta_{\text{crit}}$ as a function of the shear parameter $s$.](image3)

This work is supported by the US DOE grants DE-FG02-01ER54624 and DE-FG02-04ER54793. This research used resources of the NERSC, supported by the Office of Science, US DOE, under contract DE-AC02-05CH11231.


