Microwave beam broadening in turbulent plasma

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Introduction

Quasi-optical microwave beams possessing sufficiently small divergence are often used in fusion plasmas for the purpose of diagnostics (Doppler reflectometry [1] and collective Thomson scattering [2]) and ECRH heating and planned for application in ITER [3]. Small angular width of these beams is essential for diagnostics wavenumber resolution whereas a modest spatial width of the beam could be critical for the local ECRH power deposition. Propagation in the edge turbulent plasma of magnetic fusion devices can influence (increase) both angular and spatial width of the microwave beam leading to degradation of diagnostics resolution or broadening of the power deposition profile which attracts considerable attention nowadays [4, 5].

In the present paper the influence of turbulent plasma density fluctuations on angular and spatial beam width is treated analytically in the framework of WKB based eikonal method developed in statistical radio physics [6]. Explicit expression for the beam angular width dependence on the turbulence and plasma parameters is obtained in the case when the multiple small angle scattering dominates. Analytical expression for the microwave beam width growth along the trajectory depending on the turbulence and plasma parameters is compared to time-averaged results of 2D Maxwell's equations solver in the case of ordinary mode propagation in turbulent inhomogeneous plasma for different turbulence k-spectra and plasma conditions. Reasonable agreement of analytical and numerical treatment results is demonstrated within the domain of quasi-optical approximation validity. Significant broadening of microwave beams is predicted for future ECRH experiments at ITER. Substantial degradation of resolution of the collective Thomson scattering system under development for ITER [3] is expected as well.

Eikonal method approach

The eikonal method we are going to generalize to the case of inhomogeneous plasma came from physics of turbulent atmosphere [6]. We consider plasma in external magnetic field. Assuming that mean density distribution is one-dimensional inhomogeneous in a direction perpendicular to the external magnetic field, choose the axes in the following way: axis z is directed along the external magnetic field and axis x is directed along the density gradient. Under these assumptions we can describe the wave electric field by the Helmholtz equation

\[
\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_0 (x)^2 + h_u \frac{\omega^2 \delta n(x, y)}{c^2 n_v} E_o (x, y) = 0
\]

where \(\alpha = o, e\) stand for the ordinary and extraordinary modes, \(E_o \equiv E_o, E_e \equiv E_e\), and

\[
k_o^2 = \frac{\omega_o^2}{c^2} [1 - \nu], k_e^2 = \frac{\omega_o^2 (1-\nu)^2}{1-\nu - \nu}, \nu = \frac{\omega_{pe}}{\omega_o^2}, u = \frac{\omega_{ce}}{\omega_o^2}, h_o = 1, h_e = \frac{1-2\nu)(1-u)}{[1-u-\nu]^2}.
\]
Supposing the turbulence correlation length larger than the EC wave length we may describe the wave propagation using the WKB approach and with the accuracy to the second order in the density perturbation \( \delta n(x, y) \) assume that it influence only the phase of the wave not affecting its amplitude and the ray trajectory. Under this assumption in paraxial approximation we get the microwave spatial structure in the form

\[
E(x, y) \approx \int \frac{dk_y}{2\pi} \frac{A(k_y)}{\sqrt{k(x)}} \exp \left[ ik_y y - \frac{i}{2} k_y^2 d^2(x) + i \int k(x') dx' - i \frac{1}{2 c^2} \frac{d}{k(x)} \delta n(x', y) h(x', y') \right]
\]

where \( A(k_y) = \sqrt{k(x)} \int E(0, y) \exp(-ik_y y) dy \) stands for the antenna diagram determined by the field distribution at the plasma boundary, \( d^2(x) = \int_0^1 dx' k(x'), y' = (y, x, x', k_y) = y - k_y \int_0^1 dt / k(t) = y - k_y l^2(x') \) is the ray trajectory possessing poloidal wavenumber \( k_y \) and connecting points \( (x, y) \) and \( (x', y') \).

The microwave beam angular width is related to its poloidal structure which in its turn is characterized by the electric field two-point cross-correlation function (CCF)

\[
< E(x, y_1) E^*(x, y_2) > = \int \frac{dk_y dq_y}{(2\pi)^2} \frac{A(k_y)A(q_y)}{k(x)} \exp \left( i \phi_0 + i (\phi_1 - \phi_2) \right)
\]

where the following notation is introduced \( \phi_0 = k_y y_1 - q_y y_2 - d^2(x)(k_y^2 - q_y^2) / 2 \), \( \phi_1 = \omega / 2c^2 \int h(x', y') dx' k(x') \delta n(x', y') / n_x \), \( k_1 = k_y \), \( k_2 = q_y \). It’s natural to assume that the random phases in the exponent been integrals of independent equally distributed random values over long intervals are normally distributed due to the central limit theorem. This assumption allows performing the averaging of the exponential factor in (3) explicitly

\[
< \exp[-i(\phi_1 - \phi_2)] >= \exp[-<(\phi_1 - \phi_2)^2 > / 2].
\]

Using this relation and supposing the plasma and turbulence (statistical) inhomogeneity scale length large compared to the density fluctuations radial correlation length we can further simplify expression (3) and finally obtain in the case of strong microwave phase modulation, when \( < \phi^2 > \ll 1 \), the following expression for the electric field CCF

\[
< E(x, y_1) E^*(x, y_2) > = \int \frac{dk_y dq_y}{(2\pi)^2} \frac{A(k_y)A(q_y)}{k(x)} \exp \left( i \phi_0 - \int_0^x dx' D(x') \left( y_1 - y_2 - (k_y - q_y) l'^2 \right)^2 \right)
\]

where \( D(x) = \frac{\omega^2 h^2(x)}{c^2 k^2(x)} \int \frac{dk_y}{16\pi} \kappa_y^2 \int \frac{\delta n^2}{n_x^2} \bigg|_{k = k_y} \). In the case of Gaussian incident microwave beam \( E(0, y) \exp(-y^2 / \delta^2) \) the CCF poloidal spectrum averaged over the poloidal can be explicitly evaluated as

\[
\int dy_1 dy_2 < E(x, y_1) E^*(x, y_2) > \exp(-ik_y (y_1 - y_2)) = \exp[-k_y^2 / \sigma^2]
\]

with \( \sigma^2 = 2 / \delta^2 + 4 \int_0^x D(x') dx' \) being the angular width of the CCF poloidal spectrum. In the case of wave propagation in homogeneous plasma in which statistically homogeneous turbulence is excited the angular width of the beam squared growth linearly with the length of the trajectory \( x \). This kind of dependence is typical for the diffusive phenomena and corresponds to the photon diffusion due to the small-angle-scattering from the long scale density fluctuations. The microwave spatial width dependence on the trajectory length can
be derived using expression (4) at \( y_1 = y_2 \). The integral evaluation in this case results in relation

\[
\langle E(x,y)E^*(x,y) \rangle \sim \exp \left[ - \frac{y^2}{w^2} \right]
\]

(6)

where \( w^2 = \delta^2 / 2 + 2\delta'^2(x) / \delta^2 + 4\int_0^x dx D(x')l(x') \) is the beam spatial width with the first term being the initial beam width, the second one corresponding to a diffraction expansion and the third one describing the turbulence induced broadening.

**Comparison with numerical modeling**

In this section a comparison between analytical predictions and temporally averaged results of full-wave 2D Maxwell's equations solver 2D modeling of the ordinary mode propagation in turbulent inhomogeneous plasma is presented for two cases. In the first one the density profile is linear and turbulence is statistically homogeneous. The distributions are shown in Fig. 1a. The density fluctuations perpendicular wavenumber spectrum used in both of cases is shown on the Fig. 1b. The microwave frequency is equal to 105 GHz. The density perturbation level (fluctuation rms) is equal to 3.6% of critical density for this frequency. The phase perturbation RMS squared in this case grows linearly with the wave trajectory length and exceeds unity very close to the plasma boundary (see Fig.2a). As it is shown on the Fig.2b analytical predictions for the beam spatial broadening agree very well with numerical modeling results in this case. The second considered case is an ITER-like case. The frequency was taken equal to 170 GHz like in ECRH experiments planned in the ITER. Fluctuation and density profiles for this case are presented on the Fig. 3a. As it is seen in Fig. 3b, the phase perturbation RMS squared in this case grows quickly and exceeds unity near the maximum of the density perturbation. It seems that divergence between analytical and numerical results (Fig.3c) is arising from mistakes of the paraxial approximation on the long trajectory and can be eliminate using correct ray trajectories launched into the plasma accounting for the angular broadening in the edge region obtained using paraxial approximation. The corresponding microwave beam width is given by expression \( w(x) = w_0 + \sigma_0 \sqrt{2} \int_{x_0}^{x} dx' \sqrt{k^2(x') - \sigma_0^2} \), where \( w_0 \) and \( \sigma_0 \) are spatial and angular beam widths just after crossing the turbulence zone correspondingly and \( x_0 \) is the end of the zone. As it is seen in Fig.4c, accounting for the non-paraxial effects improves the agreement of analytical predictions and numerical results. It should be noted that this
method can be helpful also for prediction of the beam width on the long trajectory far from the edge turbulent layer, where the slab geometry is no longer applicable.

**Expected broadening for a diagnostic based on a CTS**

The collective Thomson scattering (CTS) diagnostic was installed and successfully tested on the TEXTOR tokamak and planned for application in the ITER [2]. However, different tokamak and probing wave parameters (110 GHz X-mode wave in TEXTOR and 60 GHz X-mode wave in ITER) cause different character of beam propagation in turbulent plasma. Expected angular beam broadening in the presence of homogeneous fluctuations with an amplitude equal to 0.5% of central density and Gaussian wavenumber spectrum with a correlation length equal to two hydrogen Larmor radii normalized to the initial beam width for both of tokamaks is shown on the Fig. 4a, b. A very strong broadening is predicted there for ITER.

**References**