Synchrotron radiation from runaway electron distributions in tokamaks

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Introduction Analysis of the synchrotron radiation emitted by runaway electrons (RE) in a fusion plasma presents a direct route to knowledge of the RE population, through the strong dependence of the synchrotron spectrum on particle momentum and pitch-angle. Synchrotron radiation diagnostics can thus contribute to the understanding of the processes of runaway beam formation and loss in tokamaks. Previously, synchrotron spectra have been interpreted under the assumption that all runaways have the same energy and pitch-angle [1, 2]. Here we present synchrotron radiation spectra for typical avalanching runaway distributions and compare them to the emission of mono-energetic runaways with well-defined pitch-angle. We examine the effect of including magnetic field curvature, and compare our calculations to experimental data from DIII-D.

Spectrum from runaway electron distribution The synchrotron radiation spectrum can be obtained by performing the integral

$$P(\lambda) = \frac{2\pi}{n_r} \int_{R_r} f_{RE}(p,\chi)\, \mathscr{P}(p,\chi,\lambda)\, p^2 \mathrm{d}p\, \mathrm{d}\chi \ , \tag{1}$$

where $p = \gamma v/c$ is the normalized momentum, $\chi = p_\parallel/p$ is the cosine of the pitch-angle, $\mathscr{P}$ is the single particle emission formula, f_{RE} is the runaway distribution and R_r is the runaway region of momentum space. As we normalize to n_r, $P(\lambda)$ is the average emission per runaway. The synchrotron emission at wavelength λ from a single runaway electron in straight magnetic field $\mathscr{P}_{\mathrm{cyl}} = \mathscr{P}_{\mathrm{cyl}}(\lambda, p, \chi)$ is given in Ref. [3]. In a tokamak, the effects of magnetic field line curvature have to be taken into account. This has been done in Ref. [4], resulting in the expression $\mathscr{P}_{\mathrm{full}}$. In Eqs. (21) and (26) of Ref. [4], two approximative limits of $\mathscr{P}_{\mathrm{full}}$ are given. The first of these asymptotic formulas, $\mathscr{P}_{\mathrm{as1}}$, was the expression used in runaway studies in TEXTOR [1] and DIII-D [2] and to calculate the synchrotron radiation of an avalanching population of positrons in Ref. [5]. The conditions required for validity of $\mathscr{P}_{\mathrm{as1}}$ lead to a narrow range for the wavelengths that can be considered. For wavelengths in the $\sim 0.5\mu$m range (as in the measurements described in Ref. [2]), $\mathscr{P}_{\mathrm{as1}}$ is only valid for particles with large normalized momenta p and if $v_\perp/v_\parallel$ is large ($\gtrsim 0.15$) or very small. The other formula $\mathscr{P}_{\mathrm{as2}}$ requires that

$\lambda \ll (4\pi/3)R\eta/[\gamma^3(1+\eta)^3]$ is fulfilled, where $\eta = eBRv_\perp/(\gamma m_e v_\parallel^2)$ and R is the tokamak major radius.

In large tokamak disruptions, secondary runaway generation is expected to dominate, leading to exponential growth of REs. In this case, the RE distribution can be approximated by an analytical distribution f_{AA} derived in Ref. [6]. The distribution is sensitive to the plasma parameters: parallel electric field $E_\parallel$, background electron density n_e, background plasma temperature T, and effective ion charge z_{eff}. An upper cut-off $p = p_{\text{max}}$ of the distribution must be introduced in order to avoid infinite synchrotron emission. This cut-off is physically motivated by the finite life-time of the accelerating electric field and the presence of loss mechanisms such as radiation and radial transport.

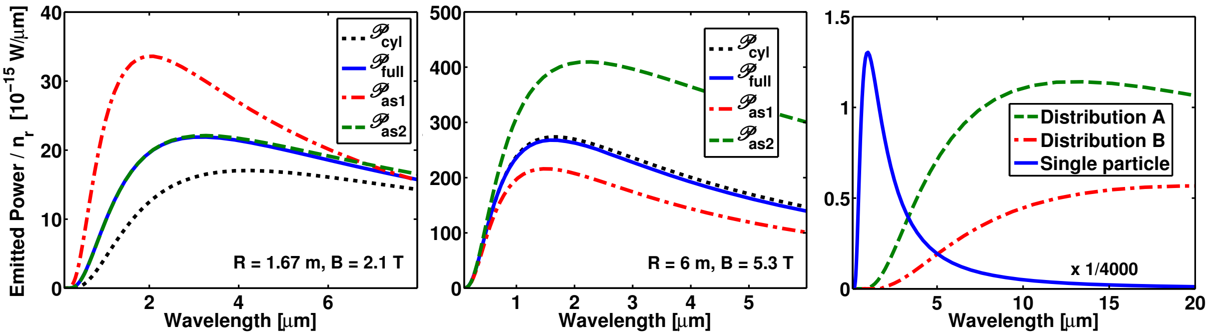

Figure 1: (a) and (b): Synchrotron spectra calculated using $\mathscr{P}_{\text{cyl}}$, $\mathscr{P}_{\text{full}}$, $\mathscr{P}_{\text{as1}}$ and $\mathscr{P}_{\text{as2}}$ in (a) DIII-D and (b) ITER. (c): Synchrotron spectra (average emission per particle) from $\mathscr{P}_{\text{cyl}}$ for two RE distributions and a single particle in a DIII-D-like scenario. The parameters are $T = 2\,\text{eV}$ and **A**: $E_\parallel = 2\,\text{V/m}$, $z_{\text{eff}} = 1$, $n_e = 5\cdot 10^{19}\,\text{m}^{-3}$, **B**: $E_\parallel = 10\,\text{V/m}$, $z_{\text{eff}} = 1.5$, $n_e = 1\cdot 10^{20}\,\text{m}^{-3}$.

Fig. 1a and b show synchrotron spectra calculated with Eq. (1) using f_{AA} and the emission formulas $\mathscr{P}_{\text{cyl}}$, $\mathscr{P}_{\text{full}}$, $\mathscr{P}_{\text{as1}}$ and $\mathscr{P}_{\text{as2}}$ in DIII-D-like and ITER-like tokamaks, respectively. The parameters used in the calculation were $p_{\text{max}} = 100$ (corresponding to a maximum runaway energy of roughly 50 MeV), $E_\parallel = 2\,\text{V/m}$, $z_{\text{eff}} = 1$, $n_e = 3\cdot 10^{20}\,\text{m}^{-3}$ and $T = 10\,\text{eV}$. The emission has a maximum for wavelenghts of a few μm. In the DIII-D-like case, $\mathscr{P}_{\text{full}}$ is well approximated by $\mathscr{P}_{\text{as2}}$. In ITER, $\mathscr{P}_{\text{cyl}}$ is a good approximation, which is expected since the field curvature is small there. These conclusions hold for a wide range of maximum RE energies. Synchrotron spectra calculated using $\mathscr{P}_{\text{cyl}}$ and $\mathscr{P}_{\text{full}}$ are generally qualitatively similar, and are also quantitatively similar for large machines. Note, that when taking into account the full distribution, the most suitable approximative emission formula may not be the one that has been used in previous work ($\mathscr{P}_{\text{as1}}$). Instead, depending on the size of the device and the actual runaway electron distribution, either

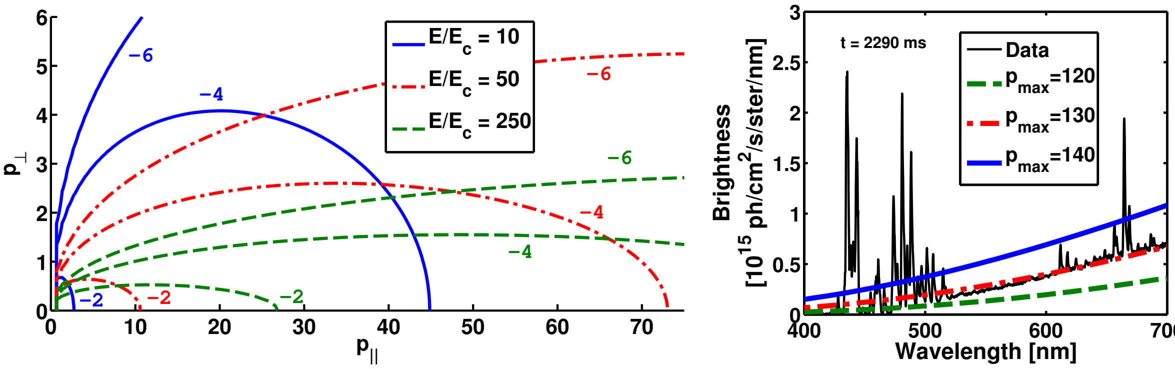

Figure 2: (a): Contour plot of $\log_{10}|f_{AA}/n_r|$, for various electric field strengths. (b): Measured visible spectrum in DIII-D fitted with theoretical spectra for various $p_{\rm max}$.

$\mathscr{P}_{\rm cyl}$ (for large devices) or $\mathscr{P}_{\rm as2}$ (for medium-sized devices) is more suitable.

In Refs. [1, 2] the synchrotron spectrum is calculated by multiplying the single particle emission $\mathscr{P}_{\rm as1}$ for specific p and χ by the number of runaways. If we take into account the whole runaway electron distribution, the spectrum changes. The single particle synchrotron emission formulas are independent of the plasma parameters, but these quantities still affect the spectrum through their influence on the shape of the runaway distribution. Figure 1c shows a comparison of the synchrotron spectrum calculated for the runaway distribution (f_{AA}) for two sets of parameters (with $p_{\rm max} = 100$) and for a single particle (with $p = 100$ and $v_\perp/v_\| = 0.15$). The figure shows that using the single-particle approximation overestimates the synchrotron emission per particle by several orders of magnitude (note that the single particle spectrum is multiplied by a small factor to fit on the same scale). The single particle approach assumes that all particles emit as much synchrotron radiation as the most strongly emitting particle in the actual distribution, which is the cause of the large discrepancy. The wavelength of peak emission is also shifted towards shorter wavelengths. Using this approximation can thus give misleading results regarding both the spectrum shape and the total emission strength.

Figure 2a shows the avalanche distribution f_{AA} in $(p_\|, p_\perp)$-space for various electric field strengths. The RE distribution, in addition to being extended in $p_\|$, becomes more narrow in $p_\perp$ as the electric field strength increases. As the most strongly emitting particles are highly energetic with large pitch-angle, the change in distribution shape will have a large impact on the spectrum. Parameter scans reveal that the average synchrotron emission increases with B, T, $z_{\rm eff}$, n_e and $p_{\rm max}$, but decreases with increasing electric field strength. The dependence on n_e and E is particularly strong, and the average emission can vary over several orders of magnitude. This variation is completely missing from the single particle approximation used previously.

Synchrotron emission in DIII-D The interest in the synchrotron emission of runaways is primarily motivated by its potential as a diagnostic tool. Here we investigate how a synchrotron spectrum from an avalanching distribution compares with an experimentally measured spectrum from DIII-D. In the specific experimental scenario we consider (shot number 146707 and time t=2290 ms, near the end of a runaway plateau phase) the loop voltage is 7 V, the plasma current is $I_p = 0.15$ MA and $n_e = 3.9 \cdot 10^{19}$ m^{-3} [7]. The runaway density can be estimated from the current given the cross-sectional area of the RE beam. The temperature is assumed to be 1.5 eV and the effective charge $Z_{\rm eff} = 1$. Conversion of the emitted synchrotron power to brightness (as detected by the DIII-D camera) can be done using Eq. (2) in Ref. [2] using the effective viewing aperture, $\theta_{\rm eff} \approx \sqrt{\theta^2 + (2/\gamma)^2 + (r_{lens}/r_0)^2}$. Here, $r_{lens} = 2$ cm is the lens aperture of the detector and $r_0 = 2$ m is the distance between the detector and the runaway beam. Since we consider the visible part of the spectrum, only particles with $p \gtrsim 50$ contribute substantially to the emission. Fig. 2b shows a comparison of spectra calculated using $\mathscr{P}_{\rm as2}$ (which is the most suitable approximation in this case) and f_{AA} for different p_{max}, and the experimentally measured spectrum, which is a superposition of synchrotron radiation from runaways and line radiation from the background plasma. The good agreement for $p_{max} = 130$ leads us to estimate the maximum runaway electron energy in this DIII-D shot to be around 65 MeV. Fitting the data with synchrotron spectra from a mono-energetic runaway population as in Ref. [2] we find a lower estimate for the runaway energy, about 40-50 MeV, depending on pitch-angle.

Conclusions Synchrotron spectra from runaway distributions can differ substantially from single-particle approximation spectra and estimating the runaway energy using the latter can be misleading. Although the single particle synchrotron emission formulas do not depend on the plasma temperature, effective charge, density or electric field strength, the total synchrotron emission is sensitive to these parameters, as these parameters determine the shape of the runaway distribution.

References

[1] R. Jaspers *et al.*, Rev. Sci. Instrum. **72**, 466 (2001).
[2] J. H. Yu *et al.*, Phys. Plasmas **20**, 042113 (2013).
[3] G. Bekefi, *Radiation Processes in Plasmas*, Wiley (1966)
[4] I. M. Pankratov, Plasma Phys. Reports **25**, 145 (1999)
[5] T. Fülöp and G. Papp, Phys. Rev. Lett., **108**, 225003 (2012).
[6] T. Fülöp *et al.*, Phys. Plasmas **13**, 062506 (2006).
[7] E.M. Hollmann *et al.*, to appear in Nucl. Fusion (2013).