Gyro-phase drift, first described by Northrop [1] is a guiding center drift that is directly dependent on the charging rate limit of dust grains. Goree et al. [2] estimated the time for a dust grain to charge up from 0 elementary charges to 1 e-fold of the equilibrium value in a homogeneous plasma as 

$$k_T T_e^{1/2}/a n_0$$,

where $T_e$ is the electron temperature in eV, $a$ is the dust grain radius in m, $n_0$ is the plasma density in m$^{-3}$, and $k_T$ is a given function of $T_i/T_e$ and $m_i/m_e$ in units of s m$^{-2}$eV$^{-1/2}$ which is fit to a numerical model. The normalized charging rate $1/\frac{dq}{dt}$ is the inverse of the charging time. The original motivation for gyro-phase drift was a complete description of all guiding center drifts of a dust grain orbiting a planet or moon. The gyro-phase drift is one of the drift motions, e.g. $E \times B$ drift, magnetic-gradient and field-line-curvature drift and grad-q drift, that results from averaging the equation of motion over the gyromotion for the dust grain in the frame that co-rotates with the planet or moon. The grad-q drift results from the gyro-angle dependent in-situ equilibrium grain charge, assuming that the grain charging is instantaneous, whereas the gyro-phase drift arises from any non-instantaneous charging modification the grad-q drift (see equation (8) of [1] and figure 9 on page 71 of [3]). The effect of introducing a gyro-phase-dependence on the grain charge leads to these two mutually perpendicular components of guiding-center drift in the absence of an electric field and its accompanying $E \times B$ drift which could be combined into a single gyro-phase drift vector. The gyro-phase drift magnitude and direction are sensitive indicators of the charging time of dust grains because of the cumulative effect of the ever-changing charge state of a grain making repeated excursions in inhomogeneous plasma. For clarity, the grad-q drift, arising from the assumption that the grain is always in its in-situ equilibrium charge state, will be labelled separately from the gyro-phase drift component that arises solely from the fact that the grain can be gyro-synchronously undercharged or overcharged with respect to its in-situ equilibrium charge state. We compute the single-grain trajectory in a dust-absent plasma using a stationary inertial lab frame. In this paper, we examine the case of an abrupt spatial inhomogeneity in UV illumination, and an adjustable charge increment parameter to highlight details of a given model’s description of charge evolution. Unlike in Northrop’s case, we examine a laboratory relevant scenario where the dust thermal speed is much smaller than the ion thermal speed. In this case, the electric field in the
plane of the gyro-motion is taken to be zero. The ion drag force, neutral drag force, gravitational force, and other typical forces on a dust grain are ignored for simplicity. The magnetic field is 4 T. We assume room-temperature ions and atoms and 1.6 eV temperature electrons for the sake of modeling the Auburn Magnetized Dusty Plasma Experiment (MDPX) [4]. The initial grain speed was oriented in the inhomogeneity direction. We also used \( n_0 = 10^{13} \text{m}^{-3} \) as the background density for \( a = 0.5 \mu \text{m} \) radius dust grains with \( T_{dust} = T[\text{Ar}^+] = 0.0025 \text{eV} \). While \( n_0 = 10^{16} \text{m}^{-3} \) would be more appropriate to MDPX, \( 10^{13} \text{m}^{-3} \) represents a simulation duration limit of our analysis.

**Numerical Approach to Charge Collection to Spherical Granule.**

A symplectic, leapfrog integrator was used to solve the equations of motion resulting from the Lorentz force. Within the larger Newton timestep, an adaptive charging step was used to ensure that only an integer number of charges were collected at a time. It is necessary to include this timestep within the larger Newton timestep because, for our choice of parameters, the dust grain will have undergone many charging timesteps during each Newton timestep. Within the charging timestep, the analytical currents for the OML charging model were applied to the dust grain. Since \( R_{Le,i} > a \) for our choice of parameters, the unmagnetized currents will be used. For \( q < 0 \), these currents are:

\[
I_e = -e4\pi a^2 n_e \frac{v_{th,e}}{2\sqrt{\pi}} \exp \left( \frac{-eq}{Ck_b T_e} \right), \quad I_i = e4\pi a^2 n_i \frac{v_{th,i}}{2\sqrt{\pi}} \left( 1 - \frac{eq}{Ck_b T_i} \right),
\]

(1)

where \( v_{th,e,i} = \sqrt{\frac{2k_b T_{e,i}}{m_{e,i}}} \), \( n_e = n_i = n_0 \), \( V_{\text{space}} - V_{\text{surface}} = q/C \), and \( C = 4\pi \varepsilon_0 a \) is the grain capacitance. With these currents specified, the charge on the grain can be updated during each charging timestep by \( q_{n+1} = q_n + \Delta q_n \), where \( \Delta q_n = \frac{I_e + I_i}{\Delta t} \). Controlling the charging rate of the dust grain is a key feature of this model, since gyro-phase drift depends on the charging rate. The charging rate for our set of parameters suggests that the grain is fully charged in \( \sim \mu \text{s} \), while the gyro-period is \( \sim 3.6 \text{s} \). For charging time much less than gyro-period, the grain charges to the in-situ equilibrium charge at each spatial location during a gyro-orbit. There should still be a grad-q drift, but no gyro-phase drift for \( a = 0.5 \mu \text{m} \), \( n_0 = 10^{13} \text{m}^{-3} \), and \( \alpha = 1 \). To arbitrarily control the grain charging and force the charging to not be instantaneous, an adjustable charging parameter, \( \alpha \), was used in the following way: \( q_{n+1} = q_n + \Delta q_n' \), where \( \Delta q'_n = \alpha \Delta q_n \). For \( \alpha = 1 \), the dust grain charges without restriction. For \( a = 0.5 \mu \text{m} \), \( n_0 > 10^{10} \text{m}^{-3} \), and \( \alpha = 1 \), the charging time is less than the gyro-period and \( \alpha < 1 \) is used to artificially delay the achievement of in-situ equilibrium charge state when plasma conditions change. If \( \alpha = 0 \), then the dust grain charge effectively never changes, no matter how large the current may be to the dust
grain. It should be noted that the numerical method does not assume any particle drifts or charge modulation of the dust grain \textit{a priori}; the integrator solves the equation of motion of the dust grain resulting from the Lorentz force during the Newtonian time step while it computes the ion and electron currents, which are both functions of the dust grain charge, \textit{i.e.}, dust grain surface potential, at each charging timestep. For the Newton time step, 2000 points/gyrocycle was used.

**Predictions of Gyro-phase Drift and Simulation Results**

Figure 1 shows a comparison of different grain trajectories for the abrupt, UV inhomogeneity for the OML model with instantaneous and non-instantaneous grain charging. All trajectories are started at \( x = 0, y = 0 \).

The dashed trajectory (left frame) is the trajectory of a dust grain that experiences a step function inhomogeneity in UV illumination (UV light exists for \( x > 0 \), while it is absent for \( x < 0 \)) and instantaneous charging (\( \alpha = 1 \)). For the portion of the grain’s gyro-orbit that it is illuminated by UV, there is a photoelectric current of electrons in addition to the plasma electron and ion currents which causes the grain charge to become less negative (instantaneously reaching this new charge state), although the dust grain surface still has a negative potential difference with respect to the local space potential, according to

\[
I_{\text{tot}} = I_e + I_i + I_{\text{ph}} \quad (2)
\]

Equation (2) describes \( I_{\text{tot}} \) when UV is present, and equation (3) describes \( I_{\text{tot}} \) when UV is absent. A smaller negative grain charge corresponds to a larger gyroradius for the grain. Each return to the shadowed region, \( (x < 0) \), cuts off the photo-electric current and the grain instantaneously recovers to the more negative charge state. This repeating pattern of abrupt modulation

![Figure 1: Dust grain Trajectories for abrupt UV inhomogeneity. The dashed, vertical line in each plot indicates the UV transition region.](image-url)
of charge state results in concatenated semi-circles which, in turn, give rise to a particle drift in the \( +y \)-direction that is mutually perpendicular to the equilibrium-charge-inhomogeneity direction and the background magnetic field. This drift is referred to as the grad-q drift. Approximating the gyroradius \( R_{L,dust} = \frac{m_{dust}v_{\perp}}{|qB|} \) to change abruptly at \( \theta = -\pi/2, 3\pi/2, ... \), which is a good assumption for \( \alpha = 1 \) and \( x_{gc} = x_0 \), the grad-q drift for this particle can be estimated by

\[
v_{\nabla q} = 2\frac{R_{L,1} - R_{L,2}}{\tau_1/2 + \tau_2/2} = 2\frac{v_{\perp}}{\pi} \frac{|q_1| - |q_2|}{|q_1| + |q_2|},
\]

where \( \tau = 2\pi m/(qB) \) is the gyro-period and the subscripts 1 and 2 indicate equilibrium values in the UV-present and UV-absent regions respectively. Knowing that the equilibrium charge in the shadowed region is \( q_1 = -1431e \pm 1e \) and the equilibrium charge in the UV region is \( q_2 = -693e \pm 1e \), the variable charge drift is predicted to be \( 8.6268 \times 10^{-4} \pm 2.335 \times 10^{-6} \text{m/s} \), which closely matches the simulation result of \( 8.640 \times 10^{-4} \text{m/s} \). The solid trajectory (Figure 1, right frame) is the resulting trajectory of a dust grain that has a step function inhomogeneity in UV illumination and non-instantaneous charging (\( \alpha = 0.001 \)). The dust grain again transitions between charge states as it goes from being shadowed (\( x < 0 \)) and illuminated (\( x > 0 \)) by UV. With non-instantaneous charging (\( \alpha = 0.001 \)), the dust grain does not immediately reach the new \textit{in-situ} equilibrium charge state, and the particle trajectory is modified. In addition to the grad-q drift, the dust grain drifts along the inhomogeneity direction (\( -x \)-direction); we refer to this drift as the gyro-phase drift. Because the gyro-center of the dust grain is drifting along the inhomogeneity direction, the dust grain samples the UV illumination region less with each gyro-orbit, causing the grain to drift out of the boundary associated with the inhomogeneity, and the guiding center drift velocity in the \( -x \)-direction decreases until it becomes zero. This drift along the inhomogeneity direction is a direct result of the finite charging rate of the dust grain in the model. With this approach, the parameter alpha permits any charging model to be evaluated in terms of gyro-phase drift and the sensitivity of the model’s predictions to non-stationary charging. Non-stationary charging has been demonstrated to be important in laboratory conditions [5].

References