Comparison between kinetic-ballooning-mode-driven turbulence and ion-temperature-gradient-driven one

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The normalized plasma pressure $\beta$ is one of the important parameters to the improvement of the magnetic plasma confinement. In the next-generation fusion devises such as JT-60SA and DEMO, high-$\beta$ plasmas are desired to realize the steady-state operation and to improve the efficiency of the fusion reactors. As $\beta$ increases, the electrostatic instabilities such as the ion temperature gradient modes (ITG) are stabilized, and the electromagnetic instabilities such as the kinetic ballooning modes (KBM) are destabilized [1]. It is important to understand how the high-$\beta$ turbulence differs from the low-$\beta$ (or electrostatic) turbulence and to predict transport levels in the high-$\beta$ regime.

We have recently extended the gyrokinetic Vlasov simulation code GKV [2] to the electromagnetic version [3]. The newly developed code enables us to simulate KBM turbulence in a high-$\beta$ regime, as well as ITG turbulence in a low-$\beta$ regime. It is found that KBM turbulence causes the smaller ion energy flux than ITG turbulence, even when they have similar values of the maximum linear growth rate. This is due to not only the smaller amplitude of the perturbations but also the smaller phase factor in KBM turbulence than those in ITG turbulence.

Turbulent fluxes, norms and phases of perturbed quantities

Employing the flux tube model, the perturbed gyrocenter distribution function $\tilde{f}_{sk}(z, v_\parallel, \mu, t)$ (where $s = i, e$ denotes ions and electrons) is represented in the perpendicular wave number space $k = (k_x, k_y)$. The GKV code solves time evolution of $\tilde{f}_{sk}$, the perturbed electrostatic potential $\tilde{\phi}_k$ and the perturbed parallel vector potential $\tilde{A}_\parallel k$ according to the $\delta f$ gyrokinetic Vlasov-Poisson-Ampère equations. The system suits for analyzing micro-instabilities and associated turbulent transport in the local limit.

The turbulent transport fluxes are evaluated by products of perturbed fields and fluid moments. For example, the energy flux caused by radial $E \times B$ flows $\tilde{v}_{Ex} = v_E \cdot \nabla x$ is given by

$$Q_{sE} = \langle \tilde{v}_{Ex}(x,y,z), \tilde{p}_s(x,y,z) \rangle = \sum_k ||\tilde{v}_{Ex,k}|| \cdot ||\tilde{p}_{s,k}|| \cdot \text{Re}[P(\tilde{v}_{Ex,k}, \tilde{p}_{s,k})] \tag{1}$$

where we define the inner product of perturbed quantities $\langle \tilde{f}, \tilde{g} \rangle \equiv \int \tilde{f} \tilde{g}^* \, dx^3 / V$, the norm $||\tilde{f}|| \equiv \sqrt{\langle \tilde{f}, \tilde{f} \rangle}$, and the phase factor $P(\tilde{f}, \tilde{g}) \equiv \langle \tilde{f}, \tilde{g} \rangle / (||\tilde{f}|| \cdot ||\tilde{g}||)$. We note that the imaginary part
of $P(\tilde{v}_{E,k}; \tilde{p}_{sk})$ does not contribute to the flux, since $Q_{SE}$ is real. The energy flux caused by magnetic perturbations can also be given by the product of perturbed magnetic fields and the parallel energy fluxes. Using $\tilde{v}_{E,k} = -ik_y\tilde{\phi}_k/B_0$, the spectrum of the energy flux is given by

$$Q_{SE,k} = \frac{k_y}{B_0} ||\tilde{\phi}_k|| ||\tilde{p}_{sk}|| \text{Im}[P(\tilde{\phi}_k; \tilde{p}_{sk})].$$ \hspace{1cm} (2)

The perturbed pressure is given by $\tilde{p}_{sk} = \int J_{0s}(0.5m_s v^2 + \mu B)\tilde{f}_{sk}dv^3$, where the zeroth-order Bessel function $J_{0s} = J_0(k_{\perp}\rho_s)$ with the perpendicular wave number $k_{\perp}$ and the Larmor radius $\rho_s$ represents the finite Larmor radius effect.

**Linear analysis of KBMs and ITGs**

Since electromagnetic effects are characterized by the normalized plasma pressure, which is now represented by the parameter $\beta_i = \mu_0n_0T_i/B_0^2$, we here show the results of linear $\beta_i$ scans. The other plasma parameters are based on the cyclone-base-case parameters [4], while the electron temperature gradient is set to be zero to stabilize the electron-temperature-gradient modes.

Figure 1 plots the linear growth rate as a function of $\beta_i$. As $\beta_i$ increases, ITGs are stabilized ($\beta_i < 0.5\%$) and KBMs are destabilized ($0.8\% < \beta_i$). We note that high-n ideal MHD ballooning modes may be unstable in high $\beta$ regime (roughly $1.4\% < \beta_i$), and that the stable regime between ITGs and KBMs ($0.5\% < \beta_i < 0.8\%$) will disappear when one employs the finite electron temperature gradient (due to the lower beta threshold of KBMs and destabilizations of the trapped electron modes).

Figure 2 shows the phase factor of the electrostatic potential and the ion pressure evaluated from the linear eigenfunctions. It is found that $\beta_i$ dependence is weak for the ITGs ($\beta_i < 0.5\%$). The phase factor is close to 1, which means that the perturbations efficiently cause...
the ion energy flux. For the KBMs ($0.8\% < \beta_i$), the phase factor increases as $\beta_i$ increases. Because of the small phase factor of KBMs, the KBMs may tend to cause ion energy flux $Q_{iE}$ smaller than ITGs even when they have the same fluctuation amplitude.

**Turbulent fluxes in KBM and ITG turbulence**

To evaluate turbulent flux in KBM and ITG turbulence, we carried out nonlinear simulations of KBM turbulence at $\beta_i = 1\%$ and ITG turbulence at $\beta_i = 0.01\%$.

Figure 3 shows the time evolution of the ion energy flux caused by the electrostatic perturbations, $Q_{iE}$, in KBM and ITG turbulence. Both cases are saturated after $tv_i/R_0 \sim 20$. Averaged over $60 < tv_i/R_0 < 160$, $Q_{iE}R_0^2/\rho_{ti}^2 n_0 T_i v_i = 15.47$ and $104.64$ for the KBM and ITG turbulence, respectively. Using the employed temperature gradient $R_0/L_{T_i} = 6.82$, diffusion coefficients are evaluated as $\chi_{iE}/\chi_{gB} = 2.27$ and $15.34$ in the gyro-Bohm unit $\chi_{gB} = v_{ti}\rho_{ti}^2/R_0$. Thus, the ion energy flux in the KBM turbulence is fifteen percent of that in the ITG turbulence even when they have similar linear growth rate.

To clarify the rationale for this difference, the spectra of the ion energy fluxes are examined. Figure 4 plots the spectrum of the norms of $||\vec{\tilde{q}}_k||$ $||\vec{\tilde{p}}_{ik}||$ summed over $k_x$ modes. The peaks at $k_y \rho_{ti} = 0.2$ are almost same values in both cases, whereas the KBM turbulence has a narrow spectrum than that in the ITG turbulence. We note that there are strong zonal modes characterized by $k_y = 0$ (not plotted here), in the ITG turbulence but not in the KBM turbulence.
Since \( k_x = 0 \) modes dominate the ion energy flux, the phase factor \( P(\tilde{\phi}_k, \tilde{p}_ik) \) for \( k_x = 0 \) modes are plotted as a function of the poloidal wave number \( k_y \) in Fig. 5. The phase factors in the KBM turbulence are smaller than those in the ITG turbulence. In terms of the most dominant \( (k_x\rho_{ti}, k_y\rho_{ti}) = (0, 0.2) \) mode, the phase factors are 0.278 and 0.857 in the KBM and ITG turbulence, respectively, which are 63% and 95% of the linear modes (see also Fig. 2). Thus, the nonlinear phase matching (i.e., the reduction of the phase factor \( P \)) occurs in the KBM turbulence.

Summary

We carried out nonlinear simulations of KBM turbulence at \( \beta_i = 1\% \) and ITG turbulence at \( \beta_i = 0.01\% \). Defining the turbulent fluxes as the products of the norms and the phase factor of the perturbations, we compared the spectra of the ion energy fluxes. It is found that KBM turbulence causes smaller ion energy flux than ITG turbulence, even when they have similar values of the maximum linear growth rate. The small ion energy flux in KBM turbulence is due to the narrow spectrum of the norms and to the small phase factor, which is reduced from that in the linear KBMs by the nonlinear phase matching.

References