FOOTPRINT OF PLASMA PARTICLES IN THE DIII-D WITH MAGNETIC PERTURBATIONS

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ABSTRACT: The symplectic DIII-D forward map [1,2] is used to calculate the footprint of the plasma particles in the DIII-D under different kind of magnetic perturbations. For a given magnetic perturbation, the last good surface is calculated using the symplectic DIII-D map for field lines. An equilibrium magnetic surface inside the last good surface is calculated from the equilibrium generating function for the field line trajectories in the DIII-D. Plasma particles are placed on this surface with uniform poloidal distribution. Particles follow the symplectic field line trajectories. Whenever a particle crosses the equatorial plane of the DIII-D, the radial expansion operator is applied: \[ R \rightarrow (1+rD)R, \] where \( R \) is radial cylindrical coordinate of the particle, \( D \) is a radial expansion coefficient and \( r \) is a random number in the interval 0 to 1. Footprints are calculated in the limit as \( D \rightarrow 0 \) and are compared with the magnetic footprint in the DIII-D. Changes in the physical parameters associated with the footprint with \( D \) are calculated. This work is supported by the US DOE grants DE-FG02-01ER54624 and DE-FG02-04ER54793. This research used resources of the NERSC, supported by the Office of Science, US DOE, under contract DE-AC02-05CH11231.

Magnetic field lines are the trajectories of a 1½ degree of freedom Hamiltonians. Tokamak plasmas are confined on the closed magnetic surfaces, and flow to the divertor on open surfaces. Closed and open surfaces are separated by the separatrix surface. Hamiltonian trajectories near the X or hyperbolic singular point of a separatrix form homoclinic tangle. For the first time we have calculated the accurate and realistic homoclinic and heteroclinic tangles of a real tokamak that preserve the symplectic topological invariance. An analytic equilibrium generating function (EGF) for the DIII-D tokamak is constructed that accurately represents the exact magnetic geometry of the DIII-D [1,2]. A new set of canonical coordinates called the natural canonical coordinates (NCC) are developed. Forward and backward symplectic maps in the NCC for the field lines are constructed. Using these maps with the EGF for the DIII-D, we have for the first time explicit symplectic homoclinic and heteroclinic tangles in the DIII-D and the simple map from qualitatively different kinds of magnetic perturbations. Tangles from the peeling-ballooning modes \((30,10)+(40,10)\) with...
amplitude $10^{-4}$ representing the type I ELMs, and $(3,1)+(4,1)$ with amplitude $10^{-4}$ are shown in Figs. 1 and 2 respectively. Radial dependence of modes is ignored.

(a). (b). (c).

(d). (e).

Fig. 1. Homoclinic tangles of the ideal separatrix and heteroclinic tangles of surfaces $\chi=0.95\chi_{\text{SEP}}, 0.9\chi_{\text{SEP}}, ..., 0.7\chi_{\text{SEP}}$ in the DIII-D from modes $(30,10)+(40,10)$ with $\delta=10^{-4}$ in $\phi=0$ plane. Each equilibrium surface consists of 360K points distributed uniformly in poloidal angle $\theta$. (a) after 1 toroidal circuit, (b) 5 circuits, (c) 10 circuits, (d) enlarged view of (b) near the X-point, and (e) enlarged view of (c). $\chi_{\text{SEP}}$ is poloidal flux inside separatrix. $(R,Z)$ are cylindrical coordinates, and $(R_0,Z_0)$ is the position of magnetic axis.
The issue of practical importance is the behavior of the plasma. The electrons and ions in plasma move rapidly along the magnetic field lines and diffuse slowly across. An important question is how the homoclinic tangle of the magnetic field lines affects the location at which electron and ions diffusing out from the main plasma volume enter the divertor chamber.

To represent this effect, we consider a surface 5 cm inside the last good surface on the line from O-point to X-point. Magnetic field lines started on this surface would remain confined forever, but electrons and ions will slowly diffuse across the magnetic field lines until they reach field lines that carry them into the divertor chamber where they are

Fig. 3. Tangles of the surfaces \( \chi = 0.95 \chi_{\text{SEP}}, 0.95 \chi_{\text{SEP}}, \) and \( 0.9 \chi_{\text{SEP}} \) in the DIII-D from modes \((3,1)+(4,1)\) with \( \delta = 10^{-4} \) in \( \phi = 0 \) plane after (a) 1 turn, (b) 5 turns, (c) 10 turns, and enlarged view of homoclinic tangles of separatrix near the X-point after (d) 1 turns, (e) 5 turns, and (f) 10 turns.
neutralized. One could simulate the diffusion using Monte Carlo methods, but a far more efficient numerical procedure is to move the particles outward by a small fractional amount \( D \ll 1 \) after each iteration of the map. Every time a particle crosses the x-axis, the radial position of the particle is increased by the factor \( (1+RD) \), i.e., \( r \rightarrow (1+RD)r \), where \( R \) is a random number between 0 and 1.

The simulations were started by launching thirty-six thousand particles on the surface 5 cm inside the separatrix with a uniform spacing in poloidal angle and on the \( \phi=0 \) plane. These particles are advanced for 1000 toroidal circuits of the DIII-D using the magnetic field line map - the forward symplectic DIII-D map - modified by the outward motion given by \( D \). On the time scale of a thousand transits, even an electron can diffuse across the plasma cross section, so this is the largest number of transits that has any physical relevance. The strike points of particles are calculated using the continuous analog of the map. The inboard collector plate is located at \( x=x_{\text{PLATE}}=-0.655 \) m. The physical coordinates \((x,y)\) are connected to the NCC by \( x=\sqrt{(2\psi/B_0)}\cos(\theta) \) and \( y=\sqrt{(2\psi/B_0)}\sin(\theta) \). \( B_0 \) is the equilibrium field on the magnetic axis of the DIII-D, \( B_0=1.589 \) Tesla. We consider the perturbation \((m,n)=(3,1)+(4,1)\) with amplitude \( \delta=10^{-4} \). We show the footprints for \( D=10^{-2}, 10^{-3}, 10^{-4}, \) and \( 5\times10^{-5} \) in Fig. 3. For \( D \leq 10^{-5} \), no particle strikes the plate. As expected, in the limit as \( D \to 0 \) the plasma footprint approaches the magnetic footprint, but the effects of the homoclinic tangle remain apparent at \( D=10^{-3} \) though not at \( 10^{-2} \). When the plasma diffusion is sufficiently rapid the effects of the tangle are washed out.

![Footprint of lines starting on a surface 5 cms inside the last good surface for the modes (3,1)+(4,1) and \( \delta=10^{-4} \) for (a) \( D=10^{-2} \), (b)\( D=10^{-3} \), (c)\( D=10^{-4} \), (d)\( D=5\times10^{-5} \), and (e) the separatrix for 10 to 1000 circuits.](image)

**Fig. 3.** Footprint of lines starting on a surface 5 cms inside the last good surface for the modes \((3,1)+(4,1)\) and \( \delta=10^{-4} \) for (a) \( D=10^{-2} \), (b)\( D=10^{-3} \), (c)\( D=10^{-4} \), (d)\( D=5\times10^{-5} \), and (e) the separatrix for 10 to 1000 circuits.

**References**
