Laminar Shocks in High Power Laser Interactions

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Some recent experiments on the interaction of high power lasers with plasmas have shown evidence of shock-like structures with very high electric fields existing over very short distances[1, 2]. Proton radiography in inertial confinement fusion capsules suggest the existence of fields of more than 10^{10} Vm^{-1} over distances of the order of 10-100 nm [3]. In a more recent paper Amendt *et al.* [4] suggest that barodiffusion (ie pressure-driven diffusion) may be a possible explanation, but this does not seem to produce very short length scales. Another relevant recent paper is that of Haberberger *et al.* [5] who describe experiments in which collisionless shocks generate high energy proton beams with small energy spread. Here we show that a shock structure can be produced by having a finite ion temperature so that some ions are reflected by the potential maximum at the shock. This produces the asymmetry between the upstream and downstream sides which destroys the familiar symmetrical ion sound solitary wave. In a collisionless unmagnetized plasma the reflected ions simply travel upstream unimpeded. Early observations of electrostatic shocks[7] show the kind of structure we describe, a potential ramp followed by downstream oscillations, at low Mach numbers. Computer simulations [8] later showed shocks, with more complicated dissipative structures at higher Mach number. More recent PIC simulations by Fiúza *et al.* [9] also report more complex turbulent shocks at higher Mach numbers than the ones used in this paper. Some later work on this problem has been carried out by Smirnovskii [10, 11].

Consider collisionless ions flowing into a region where the potential increases from zero to some positive value $\phi_{\max}$. Taking the incoming ions to have a Maxwellian distribution with average velocity V the density where the potential is ϕ, normalised to the initial density of the incoming flow is $n_i(\phi,\phi_{\max})$

$$n_i = \frac{1}{\sqrt{2\pi}}\left[\int_0^{\infty} \exp\left[-\frac{\left(\sqrt{v^2+2\phi}-V\right)^2}{2}\right]dv + \int_0^{\sqrt{2(\phi_{\max}-\phi)}} \exp\left[-\frac{\left(\sqrt{v^2+2\phi}-V\right)^2}{2}\right]dv\right] \quad (1)$$

with ion velocities normalised to the thermal velocity $V_i = \sqrt{2\kappa T_i}$ and the potential to $m_i V_i^2/(Ze)$

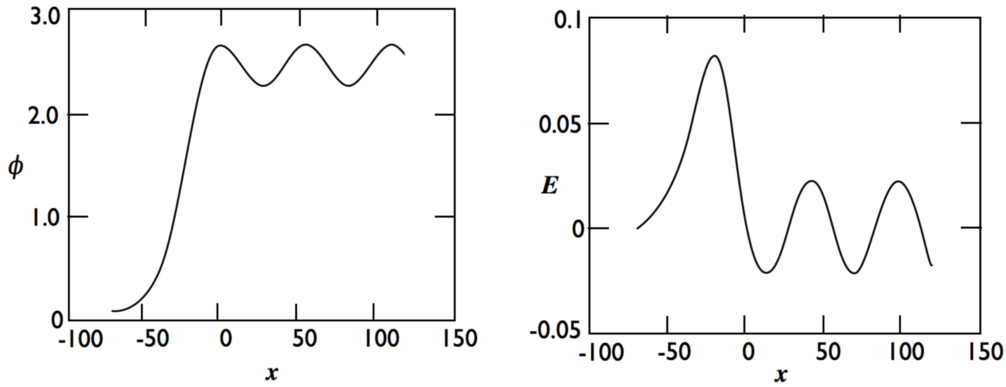

Figure 1: Left: the potential for the D-T plasma with $T_e = 20$ and $V = 4.75$. Right: the corresponding normalized electric field.

with Z the ion charge state, and $\kappa = k/m_i$ where k is Boltzmann's constant and m_i is the ion mass. We assume that V is sufficiently large that the backward part of the Maxwellian in the shock frame is negligible. The second term here takes account of particles reflected from the potential maximum. For the electrons we assume thermal equilibrium in the potential, with the electrons flowing to produce charge equilibrium far upstream where the potential tends to zero, and define $T = ZT_e/T_i$, so that

$$n_e(\phi,\phi_{\rm max}) = n_i(0,\phi_{\rm max})\exp\left(\frac{\phi}{T}\right). \tag{2}$$

We introduce the Sagdeev potential [6]

$$\Phi(\phi,\phi_{\rm max}) = \int_0^{\phi}\left[n_i(\phi',\phi_{\rm max}) - n_e(\phi',\phi_{\rm max})\right]d\phi' \tag{3}$$

so that Poisson's equation becomes analogous to the equation of motion of a particle in a potential:

$$\frac{d^2\phi}{dx^2} = -\frac{\partial\Phi}{\partial\phi}. \tag{4}$$

The condition $\Phi(\phi_{\rm max},\phi_{\rm max}) = 0$, determining $\phi_{\rm max}$, follows from the observation that the value of $\phi_{\rm max}$ must be consistent with the system dynamics. The dimensionless parameters governing the system are V and T and it can soon be found that not all combinations of these yield a system in which the Sagdeev potential has a zero for positive ϕ and is negative in the interval $(0,\phi_{\rm max})$. We have found that a value of T of around 15 or more is needed. For this value of T it appears that an acceptable solution only exists in a narrow range of Mach numbers between about 1.13 and 1.19.

If the Sagdeev potential was the same downstream of the point where the potential reaches its maximum then we would just get a standard solitary wave solution, symmetric about this

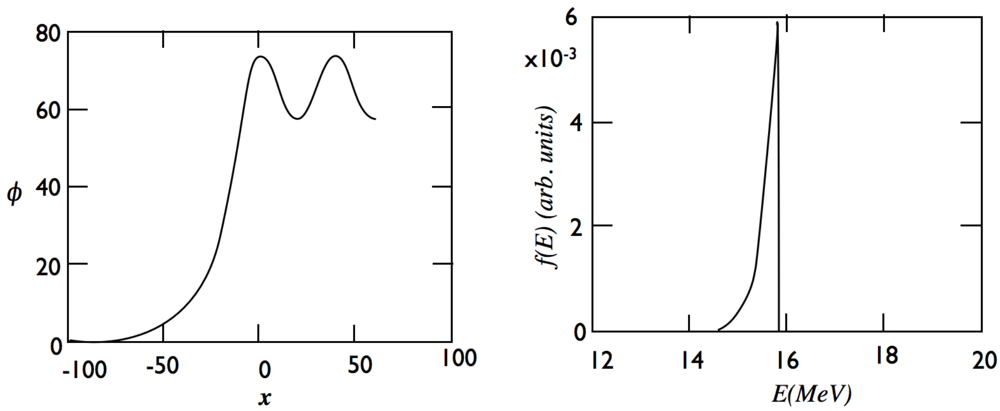

Figure 2: Left: The potential for $T_e = 100$ and Mach number 1.35. Right: The energy spectrum of reflected ions for the same parameters, including an expansion velocity of $0.1c$.

maximum. However, in the downstream region there is no reflected component and the second term in (1) is absent. This changes the Sagdeev potential and for suitable parameters produces structures with downstream oscillations.

To explore the possible relevance to a laser fusion pellet compression we can do a calculation with a 50/50 mixture of deuterium and tritium upstream. With the potential and flow speed normalised in terms of the deuterium thermal velocity the ion density is half the expression in (1) plus a corresponding tritium contribution in which ϕ is replaced with $2\phi/3$ to take account of the higher mass. For $T = 20$ and $V = 4.75$, corresponding to a Mach number of 1.06, we get the solution shown in Figure 1, left. The corresponding electric field, normalized to $m_i V_i \omega_{pi}/(Ze)$, is shown in Figure 1, right. If we assume that $Z = 1$, then we have for the electric field and length scale

$$\begin{aligned} E(V/m) &= 4.27 \times 10^{-3} E_{norm} T_i(keV)^{1/2} n_i(m^{-3})^{1/2} \\ L(m) &= 2.34 * 10^5 L_{norm} T_i(keV)^{1/2} n_i(m^{-3})^{-1/2}. \end{aligned} \tag{5}$$

If we look at the D-T result given above and assume an ion temperature of 500 eV and density 10^{28} m^{-3} then we get a peak electric field of 2.4×10^{10} V/m and, taking the normalized length of the main potential ramp to be 50, corresponding to a length of 83 nm. These parameters are in striking agreement with those quoted by Amendt *et al.* [3].

In a recent publication, Haberberger *et al.* [5] attribute the generation of ion beams well collimated in energy to a shock wave in an expanding plasma. The electron temperature they find is about an MeV and we assume that the already heated and expanding ions are at 10 keV, so that $T = 100$. The potential in this case, with a Mach number of 1.35, is shown in Figure 2, left. The normalized length scale is again about 50 which translates into a physical length of about 2

μm if we take $n = 10^{26}$ m^{-3} , while the peak electric field is around 1.3×10^{11} V/m, To compare with the experimental results, we look at the energy spectrum of the reflected ions. Adding the measured expansion velocity of $0.1c$ to the reflected ion velocity we get the spectrum shown in Figure 2, right.

This bears a striking resemblance to the experimental results, not only in the width of the spectrum and its energy but even in the detailed shape with a sharp edge on the high side. The density of reflected ions is about 8% of the background ion density, though this can go up down to less than 1% if the Mach number is reduced. The result given here appears to match the experiment much better than the computer simulation shown in the Haberberger *et al.* [5] paper. One possible explanation is that the shock in the simulation has been launched with larger Mach number of about 2. This is well above the limit beyond which our laminar solutions do not exist (around 1.4), so it may be that what is being seen is some kind of turbulent shock, producing a much broader spectrum of fast ions.

In conclusion, we have given a simple analytic description of laminar shock structures in unmagnetized plasmas and shown that the theory, despite its simplicity, can provide an explanation of results from important recent experiments on high power laser plasma interactions.

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